# Unit 9 - Rotational Motion \& Torque 

## Essential Fundamentals of Rotational Motion \& Torque

1. Torque is a twisting force that produces angular motion.

| Early E. C.: $/ 1$ |
| :--- | :--- |
| Total HW Points |
| Unit 9: $\quad / \mathbf{3 0}$ |
| Total Lab Points |
| Unit 9: $9: 40$ |
| Unit 9 Apps.: $\quad / 5$ |
| Late, Incomplete, No Work, No <br> Units Fee? $Y / N$ |

Add more here!

## Equation Sandbox

In Unit 9, some of the following equations will be used. Practice isolating variables to prepare for it.

### 9.1 Problems - Rigid Bodies, Translations, <br> and Rotations <br> Section 8.1 of your book.

1. A wheel rolls uniformly from left to right on level ground without slipping. A piece of mud on the wheel flies off when it is at the 9 o'clock position (rear of the wheel). Describe the subsequent motion of the mud.
2. A rope goes over a circular pulley with a radius of 6.5 cm . If the pulley makes 4.0 revolutions without the rope slipping:
a. What angle did the wheel turn (how many radians)?
b. What length of rope passes over the pulley?
3. A bowling ball with a radius of 15.3 cm travels down the lane so that its center of mass is moving at $3.6 \mathrm{~m} / \mathrm{s}$. The bowler estimates that it makes 7.5 complete revolutions in 2.0 seconds. Is it rolling without slipping? Prove your answer (to 2 sig figs).
4. A ball with a radius of 15 cm rolls on a level surface, and the translational speed of the center of mass is $0.25 \mathrm{~m} / \mathrm{s}$. What is the angular speed about the center of mass if the ball rolls without slipping?
5. A. The drain plug on a car's engine has been tightened to a torque of $25 \mathrm{~m} \cdot \mathrm{~N}$. If a 0.15 m long wrench is used to change the oil, what is the minimum force needed to loosen the plug?
B. In the previous problem, due to limited work space, you must crawl under the car. The force can't be supplied perpendicularly to the length of the wrench. If the applied force makes a $30.0^{\circ}$ angle with the length of the wrench, what is the force required to release the plug?
6. A. A uniform meter stick pivoted at its center has a $100.0-\mathrm{g}$ mass suspended at the $25.0-\mathrm{cm}$ position. At what point should a $75.0-\mathrm{g}$ mass be suspended to put the system in equilibrium?
B. Now remove the $75.0-\mathrm{g}$ mass from the previous problem. What mass would have to be suspended at the $90.0-\mathrm{cm}$ position for the system to be in equilibrium?

### 9.3 Problems - Rotational Dynamics <br> Section 8.3. of your book.

1. A fixed, 0.15 kg solid disk pulley with a radius of 0.75 m is acted upon by a net torque of 6.4 $\mathrm{m} \cdot \mathrm{N}$. What is the angular acceleration of the pulley?
2. What net torque is required to give a uniform $20.0-\mathrm{kg}$ solid ball with a radius of 0.35 m an angular acceleration of $10.0 \mathrm{rad} / \mathrm{s}^{2}$ ?
3. A $2200-\mathrm{kg}$ Ferris wheel accelerates from rest to an angular speed of $20.0 \mathrm{rad} / \mathrm{s}$ in 12.0 s . Approximate the Ferris wheel as a circular hoop with a radius of 30.0 m . What is the net torque on the wheel?
4. A. Two objects of different masses are joined by a light rod. If the two masses are 3.0 kg and 5.0 kg and the length of the rod is 2.0 m , find the moment of inertia about an axis of rotation through the center of the rod. (Note: use the moment of inertia equation for a point mass for each mass separately, then add them.)
B. Find the moment of inertia through the center of mass. (Note: use the moment of inertia equation for two point masses for this.) Why is this value different than in part A?

### 9.4 Problems - Rotational Kinematics

Possible 9.4 Pts: 4
Late, Incomplete, No Work, No Units Fee: $\quad-1 \quad-2 \quad-3$
Final Score: /4

## Section 7.4 of your book.

1. Circular motion review! A bike wheel rotates at a constant speed of 65 rpm during one part of a bike ride. If the radius of the wheel is 55 cm , how far does the bike travel in 1.25 minutes?
2. A. A steam engine's flywheel rotates at 450 rpm . Engineers need the wheel to speed up to 510 rpm . If the angular acceleration the engine is capable of is $5.8 \mathrm{rad} / \mathrm{s}^{2}$, how long will this acceleration take?
B. How many rotations would the wheel through during the acceleration phase?
3. A wheel spinning at 14.5 rpm comes to rest after 1450 rotations. What was the magnitude of deceleration during that time?

### 9.5 Problems - Rotational Energy \& Work Section 8.4 of your book.

1. A constant retarding torque of $12 \mathrm{~m} \bullet \mathrm{~N}$ stops a rolling wheel of diameter 0.80 m in a distance of 15 m . How much work is done by the torque?
2. A. A person opens a door by applying a $15-\mathrm{N}$ force perpendicular to it at a distance of 0.90 m from the hinges. The door is pushed wide open (to $120^{\circ}$ ) in 2.0 s . How much work was done?
B. What was the average power delivered?
3. A. A solid ball of mass 2.5 kg and radius of 0.35 m rolls along a horizontal surface with a translational speed of $5.2 \mathrm{~m} / \mathrm{s}$. How much translational kinetic energy does it have?
B. How much rotational kinetic energy does it have?
C. What percent of its total kinetic energy is translational?

# Possible 9.6 Pts: 6 <br> Late, Incomplete, No Work, No Units Fee: $\quad-1 \quad-2 \quad-3$ <br> <br> 9.6 Problems - Conservation of Angular Momentiinit <br> <br> 9.6 Problems - Conservation of Angular Momentiinit Section 8.5 of your book. 

 Section 8.5 of your book.}

1. What is the angular momentum of a $2.0-\mathrm{g}$ particle moving counterclockwise (as viewed from above) with an angular speed of $5 \pi \mathrm{rad} / \mathrm{s}$ in a horizontal circle of radius 15 cm ? Give magnitude and direction.
2. A. The Earth revolves about the Sun and spins on its axis. Assuming a circular orbit, what is the magnitude of the angular momentum associated with the Earth's orbital motion about the sun? Constants are in your Resources section (Unit 8).
B. What is the magnitude of the angular momentum associated with the Earth's rotation on its axis?
3. A. An ice skater spinning with outstretched arms has an angular speed of $1.0 \mathrm{rad} / \mathrm{s}$. She tucks her arms in, decreasing her moment of inertia to $3 / 4$ its original value. What is her resulting angular speed? Hint: This is a ratio problem, use dummy numbers for moment of inertia.
B. By what factor does her kinetic energy increase?
C. Where does the extra kinetic energy come from?

| AP Physics 1 | Unit 9.1 Lab - Torque |  |
| :---: | :---: | :---: |
| Reminder: Update Table of Contents |  | Correction Credit: <br> Half |

## Lab Overview

In small groups (6 groups maximum), build a setup that allows you to calculate the torque needed to balance a particular configuration of masses hanging from a meter stick.

## Materials

1 - Chemistry Lab Stand
Weights and Paperclips
50 Gram Mass Hanger
Customized Meter Stick
Lab Balance

## Mission 1

Use a scale to mass (in case it's irregular) a 50 gram mass plus paperclip (to hang the mass). Record this, and all other values of this mission in a well-made data table. Hang the 50 gram mass from the 30.00 cm

| Torque Lab (9.1) Scoring Guide |  |  |
| :---: | :---: | :---: |
| Contents, Titte/Date, Synopsis, Purposes |  |  |
| Mission 1: | Data Table | $/ 2$ |
|  | Net Torque | $/ 1$ |
|  | Mass at 60.00 cm | $/ 1$ |
| Mission 2: | Deflection < $5^{\circ}$ | $/ 1$ |
|  | Data Table | $/ 2$ |
|  | Exact Position: 90 g | $/ 1$ |
|  | Deflection < 5 | $/ 1$ |
| Question 1: | Mission 1 Analysis. | $/ 3$ |
| Question 2: Mission 2 Analysis | $/ 3$ |  |
| No Work Shown Fee |  | $-1-2-3$ |
| Late Lab Fee: |  | -4 |
| Total: |  | $/ \mathbf{1 6}$ | position of the meter stick (fix the stick in place so that it won't move until let go). Mass, record, and hang a 20 gram mass (and paperclip) from the 5.00 cm mark. Calculate and record the net torque provided by the masses - show all work. Calculate the mass you would need to hang at the 60.00 cm mark to perfectly counterbalance the torque produced on the other side. Using the mass hanger, rest of the weights, oddly shaped metal blobs, and any other items you find, amass enough stuff which, when suspended at the 60.00 cm mark, will make the stick remain stationary when it's released. Let me know when you have a proposed mass, then we will measure how much the ruler tips in either direction upon hanging the mass. The greater the deflection, the worse the grade!

## Mission 2

Mass, then hang the 100 g mass from the 40.00 cm mark, and the 50 g mass from the 15.00 cm mark (put all values in a data table). Calculate (show your work) the exact position you'd need to hang a weighed 90 g mass (hanger plus 40 g ) to prevent the stick from deflecting when let go. When you've calculated the position, suspend the mass and I'll let 'er rip!

## Questions:

1. What was the deflection from the first part? Do you feel that you were close, or not? Did your results surprise you? If the value was way off of level, can you account for the imbalance? Provide a thorough answer to these questions.
2. What was the deflection from the second part? Do you feel that you were close, or not? Did your results surprise you? If the value was way off of level, can you account for the imbalance?

## Lab Overview

In small groups, measure and calculate rotational parameters (radius, mass, arc length, time, angular speed, torque) that will allow you to determine the moment of inertia of a construction set flywheel. Then, determine the kinetic energy of the wheel and analyze energy transfer from a falling mass to the rotating wheel.

## Materials

1 - Base Plate $\quad 1$ - Flywheel Assembly
Elemental Weight and Hook
String with Looped Ends
Caliper Measuring Tool
Stopwatch
Lab Balance with additional masses

## Mission 1: Moment of Inertia

Report all measurements and calculations in a data table, and be sure you can read the caliper.

Moment Calculation: Using the disassembled flywheel, measure the radial and mass dimensions of the wheel to calculate the moment of inertia using the table in your Resources. Be sure to consider carefully what shape you select for each solid object, and show all calculations.

Moment Measurement: Connect a flywheel to a

| Inertia Lab (9.2) Scoring Guide |  |  |
| :---: | :---: | :---: |
| Table of Contents, Title/Date, Detailed Synopsis, Two Purposes |  | / 2 |
| Mission 1: <br> Moment of Inertia Data Table | Wheel Radius | / 0.5 |
|  | Wheel Mass | / 0.5 |
|  | Calculated Moment | / 2 |
|  | Weight's Mass | / 1 |
|  | String Length | / 1 |
|  | Average Drop Time | / 1 |
|  | Measured Moment | / 2 |
| Mission 2: <br> Energy Calculations | Angular Speed | / 0.5 |
|  | Mass' Speed | / 0.5 |
|  | $\mathrm{Ug}_{\mathrm{g}}$ | / 2 |
|  | $\mathrm{K}_{\text {trans. }}$ | / 2 |
|  | $\mathrm{K}_{\text {rot. }}$ | 12 |
| Question 1: Calculation Errors |  | / 2 |
| Question 2: Measurement Errors |  | / 2 |
| Question 3: Energy Analysis |  | / 4 |
| No Work Shown Fee |  | -1-2-3 |
| Late Lab Fee: |  | -5 |
| Total: |  | / 26 | base plate and select one of the elemental weights. Measure the mass of the weight and the length of the string, then wind the string around the flywheel (connecting one end to the notch, and the other on the hook). Time how long the mass takes to descend three times, then calculate the average angular acceleration using a kinematics equation.

## Mission 2: Energy Analysis

Using data from the previous section, calculate both the wheel's angular speed and the mass' speed at their maxima (when the weight drops off the wheel). Realize that the linear speed of the mass will equal the tangential speed of the wheel $\left(\mathrm{v}_{\mathrm{t}}=\mathrm{r} \bullet \omega\right)$. Calculate the change in gravitational potential energy ( $\mathrm{Ug}_{\mathrm{g}}$ ) of the weight, and the kinetic energies of both the moving mass ( $\mathrm{K}_{\text {trans. }}$ ) and the rotating wheel ( $\mathrm{K}_{\text {rot. }}$ ) just as the mass slips off the notch.

## Questions: Rephrase and write in complete sentences for full credit.

1. Describe sources of error of your moment of inertia calculation.
2. Describe sources of error of your moment of inertia measurement.
3. Energy Analysis: considering that the change in the mass' potential energy as it fell should equal the sum of the kinetic energies of the wheel and falling mass (excluding friction), how close did your energy calculations come to each other? What could account for differences? Finally, express the energy difference in percent form using the following equation:

$$
41 \% \text { error }=\frac{\left|U_{g}-\left(K_{\text {trans. }}+K_{\text {rot. }}\right)\right|}{U_{g}} \bullet 100 \%
$$

| AP Physics 1 | Unit 9-Rotational Motion |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Application Problems, AP Test Preparation Questions |  |  |  |  |  |
| Presentation <br> Points: | $/ 5$ | Late Fee: | -2 | Completion <br> (Booklet Check) | $/ 5$ |

Your grade on this problem set depends on the presentation you provide for your assigned problems, and whether all problems are complete when you submit your Booklet at the end of the Unit.

## Application Problems

1. A disk of radius 0.15 m rotates through $270^{\circ}$ as it travels 0.71 m . Does the disk roll without slipping? Prove your answer.
2. A worker applies a horizontal force to the top edge of a crate to get it to tip forward. If the crate has a mass of 100.0 kg and is 1.6 m tall and 0.80 m in depth and width, what is the minimum force needed to make it start to tip? Assume the center of mass of the crate is at its center and static friction great enough to prevent slipping.

3. How many different positions of stable equilibrium and unstable equilibrium are there for a cube? Consider each surface, edge, and corner to be a different position; and draw diagrams to support your answer.

## AP Test Questions

1. To loosen a bolt, a 20 N force is applied perpendicularly at the end of a 20 cm long wrench. What's the magnitude of the resulting torque?
a. $1 \mathrm{~m} \cdot \mathrm{~N}$
b. $2 \mathrm{~m} \cdot \mathrm{~N}$
c. $4 \mathrm{~m} \cdot \mathrm{~N}$
d. $10 \mathrm{~m} \cdot \mathrm{~N}$
e. $3.4 \mathrm{~m} \cdot \mathrm{~N}$
2. In the figure, what is the torque about the pendulum's suspension point produced by the weight of the bob, given that the length of the pendulum is 80 cm and $m=0.5 \mathrm{~kg}$ ?
a. $0.5 \mathrm{~N} \cdot \mathrm{~m}$
b. $1.0 \mathrm{~N} \cdot \mathrm{~m}$
c. $1.7 \mathrm{~N} \cdot \mathrm{~m}$
d. $2.0 \mathrm{~N} \cdot \mathrm{~m}$
3. A uniform $1-\mathrm{kg}$ meter stick is hanging from a thread attached to its midpoint. One block of mass $m=3 \mathrm{~kg}$ hangs from the left end fo the sitck, and another block, of unknown mass $M$ hangs below the 80 cm mark. If the stick remains at rest in the horizontal position shown, what is $M$ ?

a. 4 kg
b. 5 kg
c. 6 kg
d. 8 kg

| AP Physics 1 |  |  |  |  |  |  |  | Unit 9 Review - Rotational Motion |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Points: | $/ 20$ | Late or <br> Incomplete Fee: | $-2-4$ | -6 | Correction <br> Credit: | Final <br> Score: |  |  |  |  |  |  |

Solve these problems here, THEN enter your responses in the bubble sheet provided. Each question is worth two points.

1. A wheel rolls 5 revolutions on a horizontal surface without slipping. If the center of the wheel moves 3.2 m , what the wheel's radius?
A. 0.50 m
B. 0.30 m
C. 0.20 m
D. 0.10 m
E. 0.05 m
2. Two children are sitting on opposite ends of a uniform see-saw of negligible mass. If a $35.0-\mathrm{kg}$ child is 2.0 m from the pivot point, how far from the pivot point will her $30.0-\mathrm{kg}$ playmate have to sit on the other side for the seesaw to be in equilibrium?
A. 2.1 m
B. 2.3 m
C. 2.5 m
D. 2.6 m
E. 2.7 m
3. To start her lawnmower, Julie pulls a cord wrapped around a pulley. The pulley has a moment of inertia about its central axis of $\mathrm{I}=0.550 \mathrm{~kg} \cdot \mathrm{~m}^{2}$ and a radius of 5.00 cm . There is a frictional torque of $\tau_{\mathrm{f}}=0.430 \mathrm{~m} \cdot \mathrm{~N}$ impeding her pull. To accelerate the pulley at $\alpha=4.55 \mathrm{rad} / \mathrm{s}^{2}$, how much torque does Julie need to apply to the pulley?
A. $2.93 \mathrm{~m} \cdot \mathrm{~N}$
B. $1.22 \mathrm{~m} \cdot \mathrm{~N}$
C. $0.89 \mathrm{~m} \cdot \mathrm{~N}$
D. $0.53 \mathrm{~m} \cdot \mathrm{~N}$
E. $2.50 \mathrm{~m} \cdot \mathrm{~N}$
4. In another frictionless lawnmower (same 5.00 cm radius), that requires $3.48 \mathrm{~m} \cdot \mathrm{~N}$ of torque to pull, what is the necessary force of tension that the rope must exert?
A. 32.4 N
B. 45.6 N
C. 58.7 N
D. 66.2 N
E. 69.6 N
5. A fixed $0.23-\mathrm{kg}$ solid disk pulley with a radius of 0.10 m is acted on by a torque of $8.5 \mathrm{~m} \cdot \mathrm{~N}$. What is the angular acceleration of the pulley?
A. $7,400 \mathrm{rad} / \mathrm{s}^{2}$
B. $7,800 \mathrm{rad} / \mathrm{s}^{2}$
C. $8,200 \mathrm{rad} / \mathrm{s}^{2}$
D. $8,500 \mathrm{rad} / \mathrm{s}^{2}$
E. $8,900 \mathrm{rad} / \mathrm{s}^{2}$

## 6. New Problem Here - Choose B

 this yearA. $m \cdot N$
B.
C.
D. E.
7. A torque of $2.8 \mathrm{~m} \cdot \mathrm{~N}$ is applied to a sticky bolt. What work is done in turning the bolt 8.0 complete rotations?
A. 22 J
B. 45 J
C. 0.35 J
D. 3.6 J
E. 140 J
8. How much rotational kinetic energy does a rotating 14.9 kg hollow sphere have at 55.0 rpm ? Its radius is 2.80 m .
A. 1.18 E 5 J
B. 775 J
C. 460 J
D. 1290 J
E. 32.8 J
9. A $10.0-\mathrm{kg}$ rotating solid disk of radius 0.25 m has an angular momentum of $0.45 \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s}$. What is the angular speed of the disk?
A. $0.63 \mathrm{rad} / \mathrm{s}$
B. $0.67 \mathrm{rad} / \mathrm{s}$
C. $1.44 \mathrm{rad} / \mathrm{s}$
D. $0.81 \mathrm{rad} / \mathrm{s}$
E. $1.02 \mathrm{rad} / \mathrm{s}$
10. Which object or objects has the greater moment of inertia for the following objects: a solid disc, solid sphere, hollow sphere, hollow cylinder, or circular hoop. All objects have a mass of 15 kg , and a radius of 2.4 m . Select all that apply, there may be more than one answer.
A. Solid Disc.
B. Solid Sphere.
C. Hollow Sphere.
D. Hollow Cylinder.
E. Circular Hoop

