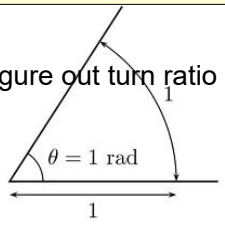
5.1 Angular Measure

Manipulatives:

gear system - figure out turn ratio based on radii values.

Side Button for Rabbit Holes!



That's RAD man!

Circles

What is the circumference of a circle?

You probably know this, but if not (AP resource):

$C = 2\pi r$ OR: $C = \pi d$	C = circumference (m) r = radius (m) d = diameter (m)

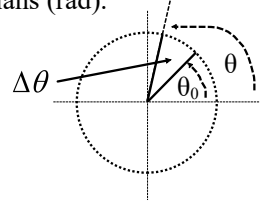
Angular Measure

When an object rotates, kinematically it is analogous to linear motion, but instead of a change in x, there's a change in angle (θ).

Angular Displacement:

$$\Delta\theta = \theta - \theta_0$$

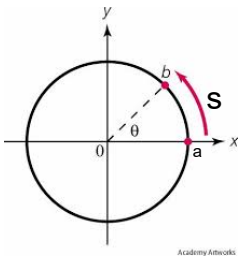
Units of θ are degrees, or radians (rad).



Arc Length

It is important to relate angular displacement of circular motion to arc length (symbol = s (not seconds!)), which is distance traveled along a circular path.

The angle θ subtends (defines) the arc length.



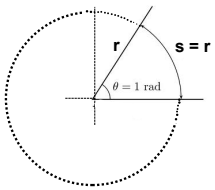
Angular Measure: Radians

The ratio of arc length to radius is the radian.

$\theta = \frac{s}{r}$	**angle = radians** **s = arc length (m)** **r = radius (m)**

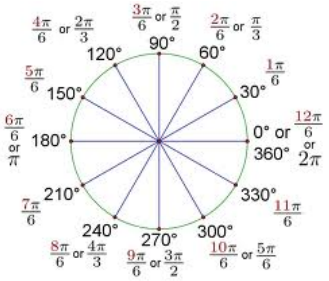
When the arc length equals the radius, the angle is one radian (1 rad (approximately 57.3^o)).

Half a circle is π radians.
A full circle is 2π radians.



Unit Circle

You probably have seen this in pre-calc, it's applicable here too.
Radians are expressed in terms of π for convenience.



Angle Conversion

Depending on situation, convert radians to degrees and vice versa:

$$Radians = Degrees \bullet \frac{\pi\, rad}{180^\circ} \qquad Degrees = Radians \bullet \frac{180^\circ}{\pi\, rad}$$

1. Convert 14.6 degrees to radians:

$$Radians = Degrees \bullet \frac{\pi\, rad}{180^\circ} = 14.6^\circ \bullet \frac{\pi\, rad}{180^\circ} = 0.255\, rad$$

2. Convert 2.3 radians to degrees:

$$Degrees = Radians \bullet \frac{180^\circ}{\pi\, rad} = 2.3\, rad \bullet \frac{180^\circ}{\pi\, rad} = 131.8^\circ$$

Arc Length Examples

A circular track has a radius of 100 m.

3. How far would you have walked if you travel a full lap?

4. What if you go 15.2 radians?

Answer 3. Full lap implies full circle: 2π radians

$$\theta = \frac{s}{r}$$

$$s = \theta \bullet r = 2\pi \bullet 100\, m = 628\, m$$

Answer 4.

$$\theta = \frac{s}{r}$$

$$s = \theta \bullet r = 15.2\, rad \bullet 100\, m = 1{,}520\, m$$

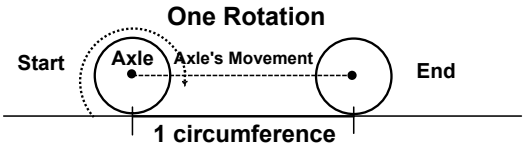
Wheel Deals

We'll be dealing a lot with wheels and gears:

Of note:

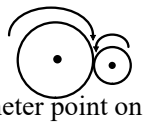
A. One full rotation (360 degrees = 2π radians): arc length described by a point on the edge = 1 circumference. Bike wheel demo.

B. One full rotation = distance axle travels.



Gear Examples

A gear (radius = 0.13 m) turns another gear (r = 0.22 m). If the small gear rotates twice:



5. What is the arc length of a perimeter point on the small gear?

$$\theta = 2\, rotations \bullet \frac{2\pi\, rad}{rotation} = 4\pi\, rad \qquad \theta = \frac{s}{r}$$

$$s = \theta \bullet r = 4\pi \bullet 0.13\, m = 1.63\, m$$

6. What is the arc length of a perimeter point of the big gear?

Since they are in contact, arc length is equal! 1.63 m

Continued Examples

7. What angle does the big gear rotate? s = 1.63 m.

$$\theta = \frac{s}{r} = \frac{1.63\, m}{0.22\, m} = 7.41\, rad$$

8. How many rotations is that?

$$rotations = \theta \bullet \frac{1\, rotation}{2\pi\, radians} = 7.41\, rad \bullet \frac{1\, rotation}{2\pi\, rad} = 1.18\, rotations$$

A note on circular ratios:

$$\frac{radius\,1}{radius\,2} = \frac{circumference\,1}{circumference\,2} = \frac{rotations\,2}{rotations\,1}$$

Inversely Related!

$$\frac{0.13\, m}{0.22\, m} = \frac{???\, rotations}{2\, rotations}$$

$$???\, rot = 0.59 \bullet 2\, rot = 1.18\, rotations$$

9. Rolling Example

A wheel (radius = 0.35 m) rolls 2.5 times around before coming to rest. How far did it go?

One full rotation is 2π radians, so 2.5 times around is 5π radians.

$$\theta = \frac{s}{r}$$

$$s = \theta \bullet r = 5\pi \bullet 0.35\, m = 5.5\, m$$

Homework 5.1

Preview 5.2

Problems 5.1 in your Booklet
Due: Next Class