5.6 Newton's Law of Uniform Gravitation

You don't understand the GRAVITY of the situation!

Gravity

An attractive force between all massive objects. Gravity holds us on this planet, and keeps planets in orbit around the sun.

It's responsible for many interactions in our universe.

So far in our pursuit of physics, we've dealt with the acceleration due to gravity (li'l g: 9.81 m/s^2).

You're about to meet li'l g's bigger brother, BIG G!

Universal Law of Gravitation

The force of gravity (F_g) is proportional to the inverse square of the distance between objects:

$$F_g \propto \frac{1}{r^2}$$

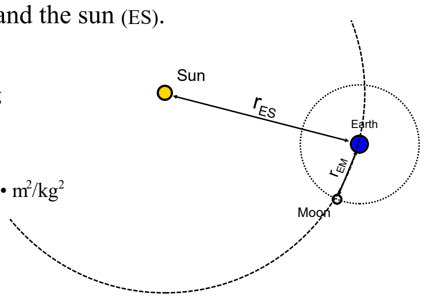
F_g can be computed between two objects:

$\left\|\vec{F}_g\right\| = \frac{G \cdot m_1 \cdot m_2}{r^2}$ **AP Equation**	**G (big G) = 6.67 E -11 N • m²/kg²** **m_1 & m_2 = masses (kg)** **r = radius (m)**

1. Earth, Moon, Sun Example

Tides are greatly affected by the moon's pull on the Earth, but what about the sun? Which has a larger gravitational pull on Earth?
Calculate F_G between the Earth and the moon (EM), and the Earth and the sun (ES).

Constants:
m_E = 6.0 E 24 kg
m_M = 7.4 E 22 kg
m_S = 2.0 E 30 kg
r_{EM} = 3.8 E 8 m
r_{ES} = 1.5 E 11 m
G = 6.67 E -11 N • m²/kg²



Earth, Moon, Sun Answer

Constants: m_E = 6.0 E 24 kg m_M = 7.4 E 22 kg m_S = 2.0 E 30 kg r_{EM} = 3.8 E 8 m r_{ES} = 1.5 E 11 m

$$F_{EM} = \frac{G \cdot m_E \cdot m_M}{r_{EM}^2} = \frac{6.67E-11Nm^2/kg^2 \cdot 6.0E24kg \cdot 7.4E22kg}{(3.8E8m)^2}$$

$$= 2.1\ E\ 20\,N\,(Earth - Moon)$$

$$F_{ES} = \frac{G \cdot m_E \cdot m_S}{r_{EM}^2} = \frac{6.67E-11N \cdot m^2/kg^2 \cdot 6.0E24kg \cdot 2.0E30kg}{(1.5E11m)^2}$$

$$= 3.6\ E\ 22\,N\,(Earth - Sun)$$

So: the sun has greater attraction. Why does the moon have such a greater effect on tides?

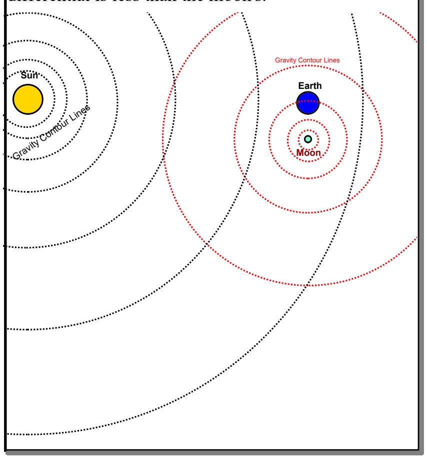
Down the Tidal Rabbit Hole!

Tides Answer Page 1

The moon's tidal effect is greater because there is a greater **differential** effect due to proximity.

Water on Earth facing the moon is relatively more affected than water on the opposite side.

The sun pulls water towards itself too, but its differential is less than the moon's.

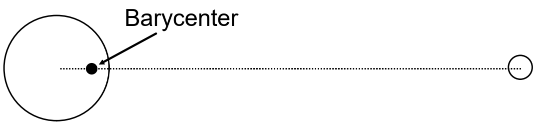


Tides Answer Page 2

There are two high tides a day. How does that work?

The lesser attraction on the far-side oceans would be expected to cause a low tide, but it doesn't.

The tide of the opposite side is caused by the barycenter (center of mass) of the Earth-Moon system being 4680 km away from the earth's barycenter.



Tides Answer Page 3

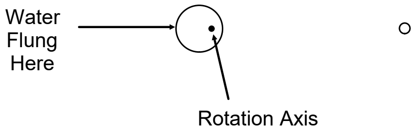
The center of rotation of the Earth-Moon system is around the barycenter, and centripetal acceleration on the far side of the planet is greater than that on the near side: Far: $a_c = BIG\ r \bullet \omega^2$ Near: $a_c = little\ r \bullet \omega^2$

F_c holding far side objects must be greater: gravity and tension are the forces doing this.

Since tension between water molecules is weak, water moves more than the land in response to this offset.

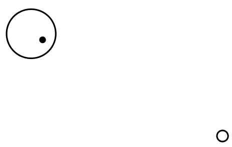
Tides Answer Pictorial

Check out the slide show! Water will be flung more on the side of the Earth farthest from the barycenter (sort of like a really big gravitron).

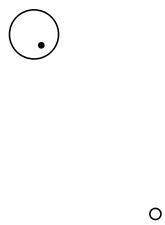


Tides Answer Pictorial

Tides Answer Pictorial



Tides Answer Pictorial



Tides Answer Pictorial

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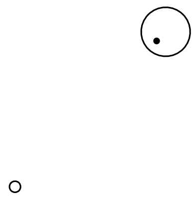

Tides Answer Pictorial

Tides Answer Pictorial

Demo: rotating pole with hanging masses.

Acceleration due to Gravity (a_g)

Attractive force diminishes with distance.

Using Newton's 2nd law of motion vs. the law of gravity, acceleration at any radius is thus:

$$F_g = m \cdot a_g \quad \text{Vs.} \quad F_g = \frac{G \cdot m \cdot M}{r^2}$$

$$m \cdot a_g = \frac{G \cdot m \cdot M}{r^2}$$

$$a_g = \frac{G \cdot M}{r^2}$$

Result: the mass accelerated is irrelevant!

$a_g = \frac{G \cdot M}{r^2}$	**G = 6.67 E -11 N • m²/kg²** **M = mass of object (kg)** **r = radius (m)**

2. Does g = a_g? Example

Calculate little g (acceleration due to gravity on Earth), using constants on Resources Page 6.

$$a_g = \frac{G \cdot M}{r^2}$$

$$= \frac{6.67E-11 N \cdot m^2/kg^2 \cdot 6.0E24 kg}{(6.4E6 m)^2} = 9.77 m/s^2$$

This is NOT 9.81 m/s^2. Have you been lied to? 9.81 m/s^2 is an average: it varies, depending on local radius (less at the poles vs. the equator), and local density. Low density areas (like oceans) vs. high density ones (like land) pull objects towards themselves differently.

Gravity at Altitude

Another way of looking at this, at some altitude (distance from Earth's surface) is:

$a_g = \dfrac{G \bullet M_E}{(R_E + h)^2}$	**G = 6.67 E -11 N•m²/kg²** **M_E = Earth's Mass = 6.0 E 24 kg** **R_E = Radius of Earth = 6.4 E 6 m** **h = altitude (m)**

Altitude Examples

On a space elevator, a 1700 kg mass is raised to 15,000 m above the earth.

3. What is its weight at that altitude?
4. How does that compare to its weight on Earth?

First, find a_g:

$$a_g = \frac{G \bullet M_E}{(R_E + h)^2} = \frac{6.67E-11 N \bullet m^2/kg^2 \bullet 6.0E24 kg}{(6.4E6 m + 15{,}000 m)^2} = 9.72 m/s^2$$

Then find its weight:

$$F_w = ma_g = 1700 kg \bullet 9.72 m/s^2 = 16{,}500 N$$

Weight on Earth:

$$F_w = mg = 1700 kg \bullet 9.81 m/s^2 = 16{,}700 N$$

5. Satellite Example

Geosynchronous satellites' orbital period equals that of one Earth day. Thus they will always be above the same point. At what altitude must one of these orbit?

Gravity serves as the centripetal force holding the satellite in place: $F_g = F_c$.
Also, the distance between the center of the Earth and the satellite is $r = R_E + h$.

Satellite Answer (1)

Setting forces equal:

$$F_g = F_c \longrightarrow F_c = m \bullet a_c = m \bullet r \bullet \omega^2$$

$$\frac{G \bullet m \bullet M_E}{r^2} = m \bullet r \bullet \omega^2$$

Isolating r: $\dfrac{GmM_E}{m\omega^2} = r^3$

Masses cancel: satellite mass is irrelevant!

$$r^3 = \frac{GM_E}{\omega^2}$$

Satellite Answer (2)

M_E = Mass of Earth = 6.0 E 24 kg

$$\omega = \frac{2\pi\, rad}{24 h \bullet \frac{3600 s}{1 h}} = 7.27 rad/s$$

$$r^3 = \frac{GM_E}{\omega^2} = \frac{6.67E-11 N \bullet m^2/kg^2 \bullet 6.0E24 kg}{(7.272E-5 rad/s)^2} = 7.57E22 m^3$$

$$r = \sqrt[3]{7.57\ E\ 22 m^3} = 4.23\ E\ 7 m$$

Finally, $r = R_E + h$, so $h = r - R_E$: (R_E = 6.4 E 6 m)

$$h = 4.23E7 m - 6.4E6 m = 3.6E7 m = 36{,}000 km$$

Homework 5.6

Preview 5.7

Problems 5.6 in your Booklet
Due: Next Class

Discuss a_c Lab