| | |
|---|---|
| **Early E. C.:** | **/ 1** |
| **Total HW Points Unit 5:** | **/ 30** |
| **Total Lab Points Unit 5:** | **/ 30** |
| **Unit 5 Apps.:** | **/ 5** |
| Late, Incomplete, No Work, No Units Fee? | Y / N |

# Unit 5 – Circular Motion & Gravitation

**Essential Fundamentals of Circular Motion & Gravitation**

1. A radian is a ratio of an arc's circumference to its diameter.
2. Right Hand Rule governs direction of rotating bodies: if the fingers of your curled right hand point in the direction of rotation, and the thumb marks the axis of rotation, if your thumb points left or towards you, the body has positive angular speed.
3. The force of gravity is inversely proportional to the square of the distance between objects.

Add More!!

**Link to Algebra**

In Unit 5, the following equations will be used. Isolate the variables indicated.

AP Equations

In-Class Equations

## Circular Motion

$Radians = Degrees \cdot \frac{\pi\ rad}{180^\circ}$ Radian/ Degree Conversion

$Degrees =$

$\theta = \frac{s}{r}$ Radian Definition

$s =$

$r =$

$\omega = \frac{\theta}{t}$ Angular Speed Definition

$\theta =$

$t =$

$\omega = \frac{2\pi}{T} = 2\pi f$ Angular Speed (Relations)

$T =$

$f =$

$a_c = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2$ Centripetal Acceleration (Relations)

$r =$

$v =$

$\omega =$

$a_t = r \bullet \alpha$ Tangential Acceleration Definition

$r =$

$\alpha =$

$\tau = r \perp F = rF \sin\theta$ Torque Equation

$r =$

$F =$

$\theta =$

$\alpha = \frac{\Delta\omega}{\Delta t}$ Angular Acceleration Definition

$\Delta\omega =$

$\Delta t =$

$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$ Angular Acceleration

$I =$

$\tau =$

$F_c = ma_c = \frac{mv^2}{r}$ Centripetal Force

$r =$

$m =$

$v =$

$v_t = r \bullet \omega$ Tangential Speed Definition

$r =$

$\omega =$

$L = I\omega$ Angular Momentum

$I =$

$\omega =$

## Gravity and Satellites

$$\left|\vec{F}_g\right| = G\frac{m_1 m_2}{r^2}$$ Force of Gravity

$r =$

$m_1 =$

$$F_g = m \bullet a_g$$ Force of Gravity

$a_g =$

$$\vec{g} = \frac{\vec{F}_g}{m}$$ Local Gravitational Constant

$\vec{F}_g =$

$m =$

$$a_g = \frac{G \bullet M}{r^2}$$ Acceleration due to Gravity

$r =$

$M =$

$$v_{esc} = \sqrt{\frac{2 \bullet G \bullet M_{obj}}{R_{obj}}}$$ Escape Velocity (Mass/Radius)

$M_{obj} =$

$R_{obj} =$

$$T^2 = K \bullet r^3$$ Kepler's Law of Periods

$T =$

$r =$

$$a_g = \frac{G \bullet M_E}{(R_E + h)^2}$$ Acceleration due to Gravity at Height

$h =$

$$v_{esc} = \sqrt{2 \bullet g \bullet R_E}$$ Escape Velocity of Earth

$R_E =$

| Possible 5.1 Pts.: 5 |
| --- |
| Late, Incomplete, No work, No Units Fee: - 1 - 2 - 3 |
| Final Score: / 5 |

## 5.1 Problems – Angular Measure
**Sections 7.1 of your book.**

1. Convert the following angles from degrees to radians:

   a. 45° b. 90° c. 120°

2. Convert the following angles from radians to degrees:

   a. $\pi/6$ rad b. $5\pi/12$ rad c. $\pi$ rad

3. How many radians are swept out by an arc length of 2.8 m for a wheel of diameter 0.34 m?

4. A wheel rolls 15 meters (therefore the arc length equals 15 meters) along flat ground. If the wheel makes 4.0 complete rotations, what is its diameter?

5. You measure the length of a distant car to be subtended by an angular distance of 1.5°. If the car is actually 5.0 m long, approximately how far away is the car?

| Possible 5.2 Pts.: 4 |
| --- |
| Late, Incomplete, No work, No Units Fee: - 1 - 2 - 3 |
| **Final Score: / 4** |

## 5.2 Problems – Angular Speed and Velocity

**Sections 7.2 of your book.**

1. A race car makes two and a half laps around a circular track in 3.0 minutes. What is the car's average angular speed?

2. A car with a 65-cm diameter wheels travels 3.0 km. How many revolutions does the wheel make in this distance?

3. The tangential speed of a particle on a rotating wheel is 3.0 m/s. If the particle is 0.20 m from the axis of rotation, how long will the particle take to make one rotation?

4. A little boy jumps onto a small merry-go-round (radius of 2.00 m) and rotates for 2.30 s through an arc length distance of 2.55 m before coming to rest. If he landed (and stayed) at a distance of 1.75 m from the central axis of rotation, what was his average angular speed and average tangential speed?

| Possible 5.3 Pts.: 4 |
| --- |
| Late, Incomplete, No work, No Units Fee: - 1 - 2 - 3 |
| **Final Score: / 4** |

## 5.3 Problems – Circular Motion & Centripetal Acceleration

**Section 7.3 of your book.**

**Wizard Challenge Alert!**

1. An Indy car with a speed of 120 km/h goes around a level, circular track with a radius of 1.00 km. What is the centripetal acceleration of the car?

2. A wheel of radius 1.5 m rotates at uniform speed. If a point on the rim of the wheel has a centripetal acceleration of 1.2 $m/s^2$, what is the point's tangential speed?

3. A gear turns one complete rotation every 2.4 seconds. If its radius is 4.5 meters, what centripetal acceleration would a point on the perimeter experience?

4. Wheel A and B are turning in a machine. If wheel A is twice the diameter of B, and rotates at twice the angular speed as B, how do the centripetal accelerations of both wheels compare? Express your value in ratio form → A : B.

| Possible 5.4 Pts.: 5 |
|---|
| Late, Incomplete, No work, No Units Fee: - 1 - 2 - 3 |
| Final Score: / 5 |

## 5.4 Problems – Centripetal Force
**Section 7.3 of your book.**

1. An Indy car with a speed of 120 km/h goes around a level, circular track with a radius of 1.00 km. If the car has a mass of 780 kg, what frictional force is necessary to keep the car on the track?

2. A rotating cylinder 16 km long and 7.0 km in diameter is designed to be used as a space colony. With what angular speed must it rotate so that the residents on it will experience the same acceleration due to gravity as on Earth (9.81 $m/s^2$)?

3. An airplane pilot is going to demonstrate flying in a tight vertical circle. To ensure that she doesn't black out at the bottom of the circle, the acceleration must not exceed 4.0 g (a 'g' is the acceleration due to gravity: 9.81 $m/s^2$) if the speed of the plane is 50.0 m/s at the bottom of the circle, what is the minimum radius of the circle so that 4.0 g limit is not exceeded?

4. Two balls, A and B, are connected by separate strings to a wheel spinning horizontally (its axis of rotation is vertical).
   A. If the mass of ball A is twice that of B, but the radius of A is half that of B, how does the tension of the two strings compare?

   B. If the radius of B is shortened to match that of A, how does tension compare then?

| Possible 5.5 Pts.: 4 |
| --- |
| Late, Incomplete, No work, No Units Fee: - 1 - 2 - 3 |
| **Final Score: / 4** |

## 5.5 Problems – Angular Acceleration
**Section 7.4. of your book.**

1. A CD originally at rest reaches an angular speed of 40.0 rad/s in 5.0 s. What is the magnitude of its angular acceleration?

2. A bike wheel originally at rest reaches a speed of 75.0 rpm in 13.0 s. What is the magnitude of its angular acceleration?

3. A flywheel slows from an angular speed of 430.0 rad/s in 115 s. What is the magnitude of its angular acceleration?

4. What is the angular acceleration of a wheel that slows from 13.8 rotations per second to resting in 50.0 seconds?

| Possible 5.6 Pts.: 4 |
| --- |
| Late, Incomplete, No work, No Units Fee: - 1 - 2 - 3 |
| **Final Score: / 4** |

## 5.6 Problems – Newton's Law of Gravitation, Section 7.5 of your book. Wizard Challenge Alert!

1. From the mass and diameter of the Moon, compute the value of the acceleration due to gravity, $g_M$, at the surface of the moon. Use the Unit 5 table in the back of your booklet.

2. A 100.0-kg object is taken to a height of 300.0 km above the Earth's surface. What is the object's mass at this height? What is the object's weight at this height?

3. Two objects are placed 3.4 meters apart: object A is 4.5 E 5 kg, and object B is 1.8 E 6 kg. What is the gravitational force between the two objects? Is this greater or less than that of the force of gravity acting on both objects by the Earth? Explain

4. Two objects are in gravitational contact with each other at some distance. Object A is three times more massive than B. If the objects are separated to two times the original distance, by what factor does the gravitational force change?

| Possible 5.7 Pts.: 4 |
| --- |
| Late, Incomplete, No work, No Units Fee: - 1 - 2 - 3 |
| Final Score: / 4 |

## 5.7 Problems – Kepler's Laws
## Section 7.6 of your book.

1. The asteroid belt that lies between Mars and Jupiter may be the debris of a planet that broke apart or was not able to form as a result of Jupiter's strong gravitation. An average asteroid has a period of about 5.0 y. Approximately how far from the Sun would this 'fifth' planet have been?

2. An astronaut using a trampoline on an asteroid of radius 3.4 km, and a mass of 6.6 E 14 kg, is in danger of jumping off the asteroid. How fast would the astronaut have to jump to clear the asteroid's gravitational attraction and be lost in space?

3. How fast would the astronaut have to jump in order to launch off the surface of the Earth?

4. Consider two objects, A and B. A is five times as massive as B, but half the radius. What is the ratio of escape velocities between the two objects?

| AP Physics 1 | 5.1 Lab - Angular & Tangential Speed |
|---|---|
| Reminder: Update Table of Contents | Correction Credit: Half |

**Lab Overview**:

Working in small groups, use a motor/flywheel/strobe light apparatus to measure and calculate the angular and tangential speeds of two wheels.

| Ang. & Tang. Speed Lab (5.1) Guide | | |
|---|---|---|
| Table of Contents, Title/Date, Detailed Synopsis, Two Purposes | | / 2 |
| **Mission 1:** | Data Table: radius | /2 |
| | Angular Speed: RPM | /1 |
| | Angular Speed: rad/s | /1 |
| | Tangential Speed: | /2 |
| **Question 1:** Improvements | | /2 |
| **Question 2:** Rotations Calculation | | /2 |
| No Calculations Fee | | -1 -2 |
| Late Lab Fee | | -3 |
| **Total:** | | **/ 12** |

**Equipment:**

Voltage Generator
Black/Red Wires
Base Plate
12 V Motor with Pulley
Gear Belt
Belt Driven Flywheel
Standardized Rulers

**Setup** – Look at the example at the front of the room for guidance.

The voltage generator should be off until you are ready to start taking data. Before plugging it in and turning it on, rotate the silver 'AC/DC VOLTAGE ADJUST' knob all the way counterclockwise, be sure the blue 0 – 24 V DC button is **in**, and be sure that the blue Amps/Volts button is **out**.

Connect the motor and flywheel to the base plate, and connect them with the belt. If the belt is loose, rotate the flywheel 180° and try again. Use the wires to connect the voltage generator to the 12 V motor – the colors don't matter in a direct current setup.

Be sure the strobe light is off before plugging it in, and be sure it is set on the low range (red button **in**). Turn the coarse adjust knob all the way counterclockwise.

**Mission 1: Flywheel and Pulley Measurements (8 Points):**

Measure and record the radius of the flywheel and motor pulley. Make marks on the pulley and flywheel (see example setup) to help gather data. Turn the voltage generator on, and turn the knob slowly until the display is set between 2.0 and 2.5 volts. The voltage will change as the flywheel increases rotation speed, so be sure that you check the voltage after a minute or two. If you apply too much voltage, you will trip the internal circuit breaker.

The flywheel will take a couple minutes to come up to speed. After it is going at a steady rate, turn on the strobe light. Slowly adjust the coarse knob until you see your markings (on either the pulley or flywheel) appear to be stationary. Your RPM value (on the strobe light readout) should be between 475 and 850; record this data.

Convert your RPM value into radians per second, then calculate the tangential velocity of a point on the edge of each of the wheels. Show all equations and work for full credit.

Make a data table that reports the radii of your two wheels, the angular speed of the system (in RPM and radians/s), and the tangential speeds of the two wheels.

**Questions: Rephrase and answer in complete sentences for full credit.**

1. What are some sources of error in Mission 1?

2. If your flywheel turned for 144 seconds, how many rotations would it have gone through?

| AP Physics 1 | 5.2 Lab - Centripetal Acceleration & Force |
|---|---|
| Reminder: Update Table of Contents | Correction Credit: Half |

**Lab Overview**:

Working in small groups, take measurements of a system in rotational motion to determine centripetal force, period, centripetal acceleration, and tangential speed at two different radii.

| Centripetal Force Lab (5.2) Guide | | |
|---|---|---|
| Table of Contents, Title/Date, Detailed Synopsis, Two Purposes | | / 2 |
| Mission 1: | Data Table: stopper | / 1 |
| | hanger/mass | / 1 |
| | Angle | / 1 |
| | Time | / 1 |
| | Angular Speed | / 2 |
| | $a_C$ | / 2 |
| | $F_C$ | / 2 |
| **Question 1:** Improvements | | / 2 |
| **Question 2:** $a_C$ comparison | | / 2 |
| **Question 3:** $F_C$ comparison | | / 2 |
| **Question 4:** Force comparison | | / 2 |
| No Calculations Fee | | -1 -2 |
| Late Lab Fee | | -4 |
| **Total:** | | **/ 18** |

**Caution:**

This lab involves swinging rubber stoppers in a wide arc that could cause damage to a person getting hit by the stopper.

**Equipment:**

Rubber Stopper
Looped String
Plastic Tube
50 g Mass Hanger
200 g Mass
Meter Stick
Stopwatch
Masking Tape
Alligator Clips

**Mission 1 (13 Points):**

Record mass data for the rubber stopper, hanger, and 200 g mass (in case it's inaccurate).

Take the string and thread it through the rubber stopper, then turn the string back on itself and through the loop at the end. Take the other (non-stoppered) end of the string and thread it through the plastic tube. Connect the 200 gram mass to the hanger and suspend it from the loop, taping the mass so it can't slide off the hanger.

Measure a point 25 cm from the end of the rubber stopper to the tube that the string goes through, then place the clip about 1 cm down from the other (hanger side) end of the tube. This will serve as a guide to stay at when you are collecting data.

Swing the stopper around until the rope is at the point you marked. Once your system is stable (no going up or down), have another student start a stopwatch while counting 20 complete revolutions. This amount will enable you to have a better average value of angular speed.

Once you have gathered data at a radius of 25 cm, measure out a radius of 50 cm, mark it, and repeat the process of determining angular speed.

Using the angular speed for both radii separately, calculate the centripetal acceleration and centripetal force for both trials. Show all work for full credit (2 Points).

Create a data table reporting the masses of the stopper, hanger + 200g mass, angle of 20 rotations, time of 20 rotations, angular speed, centripetal acceleration ($a_c$) and centripetal force ($F_c$).

**Questions: Rephrase and answer in complete sentences for full credit.**

1. What are some sources of error in Mission 1?

2. How did the two values of centripetal acceleration compare? Should they be the same, or different? Explain why.

3. How did the two values of centripetal force compare? Should they be the same, or different? Explain why.

4. How does the force of gravity compare to the centripetal force of tension?

| AP Physics 1 | Unit 5 - Circular Motion | | | | |
|---|---|---|---|---|---|
| Application Problems, AP Test Preparation Questions | | | | | |
| Presentation Points: | / 5 | Late Fee: | -2 | Completion (Booklet Check) | / 5 |

Your grade on this problem set will depend on the presentation you provide for your problem, and whether they are complete when you submit your Booklet at the end of the Unit.

1. The hour, minute, and second hands on a clock are 0.25 m, 0.30 m, and 0.35 m respectively. What are the distances traveled by the tips of the hands in a 30.0-minute interval?

2. A merry-go-round makes 24 revolutions in a 3.0-minute ride. What is the angular speed in rad/s? What are the tangential speeds of two people 4.0 m and 5.0 m from the axis of rotation?

3. Imagine that you swing a ball about your head on the end of the string. Assume the ball moves at a constant speed in a horizontal circle. If the mass of the ball is 0.250 kg, the radius of the circle is 1.50 m, and the period is 1.20 s, what is the ball's tangential speed?

4. For the previous problem, what centripetal force are you imparting to the ball via the string?

## Unit 5 Practice AP Multiple Choice Questions

The following problems (multiple choice and free response) are designed to train you to take the AP Physics 1 test in the spring, and will be scored at the end of the Unit – based on completion and accuracy.

1. An object moves at constant speed in a circular path. Which two statements are true?
    a. The velocity is changing.
    b. The velocity is constant.
    c. The magnitude of acceleration is constant.
    d. The magnitude of acceleration is changing.

2. An object moves at a constant speed in a circular path of radius r at a rate of 1 revolution per second. What is its centripetal acceleration?

   a. 0 b. $2\pi^2 r$ c. $2\pi^2 r^2$ d. $4\pi^2 r$

3. If the distance between two point particles is doubled, then the gravitational force between them
   a. decreases by a factor of 4.
   b. decreases by a factor of 2.
   c. increases by a factor of 2.
   d. increases by a factor of 4.

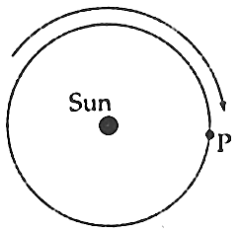
4. At the surface of the Earth, an object of mass *m* has weight *w*. If this object is transported to an altitude twice the radius of Earth, then at the new location,
   a. its mass is *m* and its weight is *w*/2.
   b. its mass is *m*/2 and its weight is *w*/4.
   c. its mass is *m* and its weight is *w*/4.
   d. its mass is *m* and its weight is *w*/9.

5. A moon of mass *m* orbits a planet of mass 100*m*. Let the strength of the gravitational force exerted by the planet on the moon be denoted by $F_1$, and let the strength of the gravitational force exerted by the moon on the planet be $F_2$. Which of the following is true?

   a. $F_1 = 100\ F_2$ b. $F_1 = 10\ F_2$ c. $F_1 = F_2$ d. $F_2 = 10\ F_1$

6. A moon of Jupiter has a nearly circular orbit of radius *R* and an orbital period of *T*. Which of the following expressions gives the mass of Jupiter?

   a. $4\pi^2 R/T^2$ b. $2\pi R^3/(GT^2)$ c. $4\pi R^2/(GT^2)$ d. $4\pi^2 R^3/(GT^2)$




7. You are looking at a top view of a planet orbiting the Sun in a clockwise direction. Which of the following would describe the velocity, acceleration, and force acting on the planet due to the Sun's pull at point P?

   a. v ↓ a ↑ F ↑ b. v ↓ a ← F ← c. v ↓ a → F → d. v ↑ a ← F ←

8. Which of the following statements are true for a satellite in outer space orbiting the Earth in uniform circular motion? Select two answers.
    a. There are no forces acting on the satellite.
    b. The force of gravity is the only force acting on the satellite.
    c. The force of gravity is balanced by outward force of the object.
    d. The mass of the satellite has no effect on the orbital speed.

## Unit 5 Example AP Question – Free Response

Show calculations for full credit.

1. The Earth has a mass of 6 E 24 kg and orbits the Sun in 3.15 E 7 seconds at a constant circular distance of 1.5 E 11 m.
    a. What is the Earth's centripetal acceleration around the sun?

    b. What is the gravitational force acting between the Sun and Earth?

    c. What is the mass of the Sun?

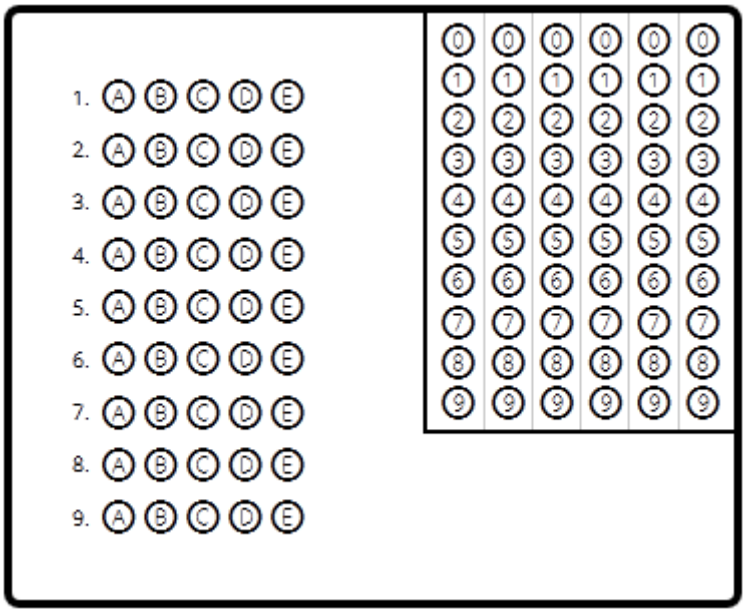
2. A curved section of a highway has a radius of curvature of *r*. The coefficient of friction between standard automobile tires and the surface of the highway is $\mu_s$.
    a. Draw and label all the forces acting on a car of mass *m* traveling along this curved part of the highway.

    b. Compute the maximum speed with which a car of mass *m* could make if around the turn without skidding in terms of $\mu_s$, *r*, *g*, and *m*.

| AP Physics 1 | | Unit 5 Review - Circular Motion | | | | | |
|---|---|---|---|---|---|---|---|
| Points: | / 18 | Late or Incomplete Fee: | -2 -4 -6 | Correction Credit: | | Final Score: | |

Solve these problems here, THEN enter your responses in the bubble sheet provided.

Each question is worth two points.




1. Two gear wheels with interlocking teeth have radii of 25 cm and 60 cm. How many radians does the smaller wheel turn when the larger wheel turns 4.0 revolutions?

A) 60. radians B) 25 radians
C) 30. radians D) 75 radians
E) 14 radians

2. A skater spins through 7.50 revolutions with her arms fully outstretched at right angles to her body. If her arms are 60.0 cm long, through what arc length distance do the tips of her fingers move?

A) 14.9 m B) 21.9 m C) 28.3 m D) 30.8 m E) 41.9 m

3. The driver of a car sets the cruise control and ties the steering wheel so that the car travels at a uniform speed of 15 m/s in a circle of diameter 120 m. What arc length does the car travel during the 4.00 minutes?

A) 1500 m B) 2200 m C) 3100 m D) 3600 m E) 3900 m

4. For the previous problem, what angular distance does the car move in 4.00 minutes?

A) 88 radians B) 60. radians C) 3.4 radians D) 12.8 radians E) 48 radians

5. A car with a constant speed of 83.0 km/h enters a circular flat curve with a radius of curvature of 0.400 km. What centripetal acceleration would the friction of the tires have to supply so the car doesn't slip off the road?

A) 0.96 $m/s^2$ B) 1.19 $m/s^2$ C) 1.33 $m/s^2$ D) 1.55 $m/s^2$ E) 1.62 $m/s^2$

6. Two objects are attracted to each other with a certain gravitational force. If the original force between the objects is 0.90 N and the distance is tripled, what is the new gravitational force between the objects?

A) 0.5 N B) 2.7 N C) 0.30 N D) 0.10 N E) 1.5 N

7. Two objects with masses of 1.4 E 10 kg and 9.8 E 7 kg are 508 m apart. What is their gravitational attractive force?

A) 410 N B) 398 N C) 355 N D) 325 N E) 278 N

8. An alien spacecraft is in orbit around our sun, and it has a period of 4.259 years. How far away from the sun is the spacecraft?

A) 3.93 E 8 km B) 8.99 E 8 km C) 1.05 E 9 km D) 5.64 E 9 km E) 4.29 E 9 km

9. A steel sphere (mass 2.8 kg) on the end of a rope swings in a circle with a radius of 1.4 m. If the tension on the rope can't exceed 25 N, what is the maximum tangential speed at which the sphere can rotate?

A) 2.8 m/s B) 3.5 m/s C) 6.4 m/s D) 8.8 m/s E) 12.5 m/s