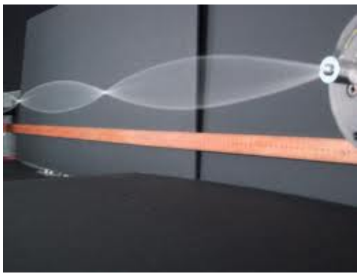
12.2 - Standing Waves and Resonance



Standing Waves

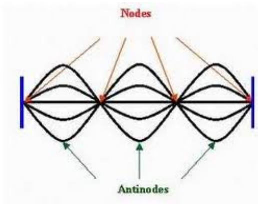
Interference between reflected waves of some frequency leads to a steady waveform. (String demo.)

Node: Stationary part of a standing wave.

Antinode: Points of maximum amplitude of a standing wave.

The spacing between nodes or antinodes is half a wavelength:

$$\frac{\lambda}{2}$$

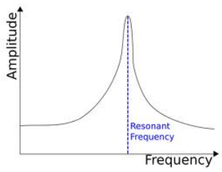


Resonant Frequencies

A natural frequency of vibration, at which standing waves are produced.
This depends on mass, elasticity or restoring force, and geometry (length, width, etc).

All oscillating systems have at least one resonant frequency, the lowest is called the fundamental frequency (f_1).

Sonic Response Graph

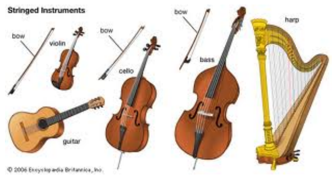


Stretched Strings

A string stretched between points vibrates when plucked, struck, or bowed.

Each end of the string is a node, and an integral number of *half*-wavelengths exists between nodes.

The lowest frequency possible in a harmonic system is called the fundamental frequency (denoted f_1).



Oscillation Frequencies

Possible string frequencies are defined thusly:

$f_n = \frac{v}{\lambda_n}$ $= n\left(\frac{v}{2L}\right)$ $= nf_1$	f = frequency (Hz or s^{-1}) n = harmonic number (1, 2, 3, . . .) v = wave speed on the string (m/s) λ = wavelength at n (m) L = length of string (m) f_1 = fundamental frequency (Hz)

Easy Way

Other Parameters

Wave speed (v) depends on string mass and tension:

$v = \sqrt{\frac{F_T}{\mu}}$	v = wave speed (m/s) F_T = tension of string (N)
	μ = linear mass density (kg/m) $\mu = \frac{string's\ mass}{length\ of\ string}$

Pulling it Together

Substituting for velocity, we get an equation that governs any string:

$$f_n = \frac{n}{2L}\sqrt{\frac{F_T}{\mu}} = nf_1$$

1. Frequency Example A

A 1.15 m string has a mass of 20.0 g (0.020 kg), and tension of 6.3 E 3 N.
What's the string's fundamental frequency (f_1)?

First, find μ:

$$\mu = \frac{string's\ mass}{length\ of\ string} = \frac{0.020kg}{1.15m} = 0.0174\,kg/m$$

Then compute f_1:

$$f_1 = \frac{n}{2L}\sqrt{\frac{F_T}{\mu}} = \frac{1}{2 \bullet 1.15m}\sqrt{\frac{6.3E3N}{0.0174\,kg/m}} = 262Hz$$

2. Frequency Example B

What frequencies are the next two harmonics (f_2 & f_3)? f_1 was 262 Hz.

2nd Harmonic:

$$f_n = nf_1$$

$$f_2 = 2(262Hz) = 524Hz$$

3rd Harmonic:

$$f_3 = 3(262Hz) = 786Hz$$

Resonance

When an oscillating system is driven at one of its natural frequencies, a maximum amount of energy is transferred.

Consider a swing: pushing the swing in time with its motion makes it go higher. If you're out of sync, it doesn't.

Resonance Disasters

When the frequency of a driving force matches a system's harmonics, amazing results happen!

You've all heard of the urban legend that a glass can be shattered with a human voice? It's no legend. http://www.youtube.com/watch?v=l4jdGf3RzCs

In 1831, a bridge in England collapsed when soldiers marching in step matched one of its harmonic frequencies. Now, soldiers break step.

Wind affects things too!

http://www.youtube.com/watch?v=zpUL6sZs6J4

Resonance Field Trip

An important life lesson awaits you in the elevator at the north side of the school (maybe).

Homework

12.2 Problems.
Due: Next Class.