12.2 - Standing Waves and Resonance

Standing Waves
Interference between reflected waves of some frequency leads to a steady waveform. (String demo.)
Node: Stationary part of a standing wave.
Antinode: Points of maximum amplitude of a standing wave.
The spacing between nodes or antinodes is half a wavelength:
$$\frac{\lambda}{2}$$

Resonant Frequencies
A natural frequency of vibration, at which standing waves are produced.
This depends on mass, elasticity or restoring force, and geometry (length, width, etc).
All oscillating systems have at least one resonant frequency, the lowest is called the fundamental frequency ($f_1$).

Stretched Strings
A string stretched between points vibrates when plucked, struck, or bowed.
Each end of the string is a node, and an integral number of half-wavelengths exists between nodes.
The lowest frequency possible in a harmonic system is called the fundamental frequency (denoted $f_1$).

Oscillation Frequencies
Possible string frequencies are defined thusly:
$$f_n = \frac{v}{\lambda_n} = n\left(\frac{v}{2L}\right) = nf_1$$
$f$ = frequency (Hz or s$^{-1}$)
$n$ = harmonic number (1, 2, 3, . . .)
$v$ = wave speed on the string (m/s)
$\lambda$ = wavelength at n (m)
$L$ = length of string (m)
$f_1$ = fundamental frequency (Hz)

Other Parameters
Wave speed ($v$) depends on string mass and tension:
$$v = \sqrt{\frac{F_T}{\mu}}$$
$v$ = wave speed (m/s)
$F_T$ = tension of string (N)
$\mu$ = linear mass density (kg/m)
$$\mu = \frac{\text{string's mass}}{\text{length of string}}$$
Pulling it Together
Substituting for velocity, we get an equation that governs any string:

\[ f_n = \frac{n}{2L} \sqrt{\frac{F_T}{\mu}} = nf_1 \]

1. Frequency Example A
A 1.15 m string has a mass of 20.0 g (0.020 kg), and tension of 6.3 E 3 N.
What's the string's fundamental frequency \( f_1 \)?
First, find \( \mu \):

\[ \mu = \frac{\text{string's mass}}{\text{length of string}} = \frac{0.020 \text{kg}}{1.15 \text{m}} = 0.0174 \text{kg/m} \]

Then compute \( f_1 \):

\[ f_1 = \frac{n}{2L} \sqrt{\frac{F_T}{\mu}} = \frac{1}{2 \times 1.15 \text{m} \sqrt{\frac{6.3 \times 10^-3 \text{N}}{0.0174 \text{kg/m}}}} = 262 \text{Hz} \]

2. Frequency Example B
What frequencies are the next two harmonics \( f_2 \) & \( f_3 \)?
\( f_1 \) was 262 Hz.
2nd Harmonic:

\[ f_2 = nf_1 = 2 \times 262 \text{Hz} = 524 \text{Hz} \]

3rd Harmonic:

\[ f_3 = 3f_1 = 3 \times 262 \text{Hz} = 786 \text{Hz} \]

Resonance
When an oscillating system is driven at one of its natural frequencies, a maximum amount of energy is transferred.
Consider a swing: pushing the swing in time with its motion makes it go higher. If you're out of sync, it doesn't.

Resonance Disasters
When the frequency of a driving force matches a system's harmonics, amazing results happen!
You've all heard of the urban legend that a glass can be shattered with a human voice?
It's no legend. http://www.youtube.com/watch?v=I4jdGf3RzCs
In 1831, a bridge in England collapsed when soldiers marching in step matched one of its harmonic frequencies. Now, soldiers break step.
Wind affects things too!
http://www.youtube.com/watch?v=rzqJULsZs5J4

Resonance Field Trip
An important life lesson awaits you in the elevator at the north side of the school (maybe).

Homework
12.2 Problems.
Due: Next Class.