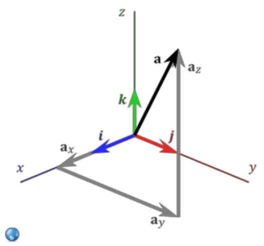
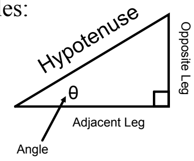
3.1 Vector Manipulation



Trig Review

This is in your AP Test Resources if you need it.

Remember the relations for right triangles:



$S = \frac{O}{H}$: $\sin\theta = \frac{Opposite}{Hypotenuse}$

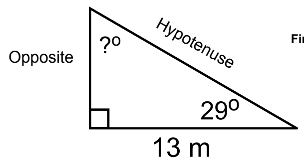
$C = \frac{A}{H}$: $\cos\theta = \frac{Adjacent}{Hypotenuse}$

$T = \frac{O}{A}$: $\tan\theta = \frac{Opposite}{Adjacent}$

Read: "The sine of angle theta is equal to the ratio of the opposite leg over the hypotenuse."

Trig Example: BE IN DEGREES!!

Find the missing sides and angles for the following:



Find Hypotenuse:

$C = \frac{A}{H}$: $\cos\theta = \frac{Adjacent}{Hypotenuse}$

$Hypotenuse = \frac{Adjacent}{\cos\theta}$

$Hypotenuse = \frac{13\,m}{\cos 29^\circ} = 14.86\,m$

Find Opposite side: $T = \frac{O}{A}$: $\tan\theta = \frac{Opposite}{Adjacent}$

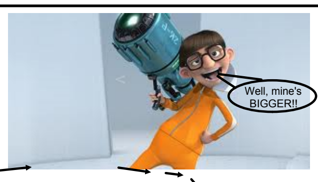
$Adjacent \bullet \tan\theta = Opposite$

$13\,m \bullet \tan 29^\circ = 7.21\,m$

Find third angle: the sum of all interior angles of a triangle = 180º.

90º + 29º + ?º = 180º ? = 61º

Vectors



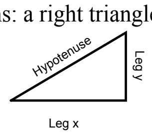
Notation: Represented by an arrow above a variable:

$\vec{A}$ means: "Vector A"

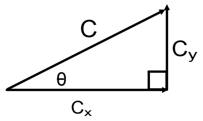
Vectors have x and y components:
Often written: <x, y>.

Resolving (Decomposing) Vectors

Based on trigonometry relations: a right triangle's hypotenuse is the vector sum of its legs.



Given a vector's magnitude and direction, decompose it into its x and y components:



Ex: Vector C is composed of C_x and C_y

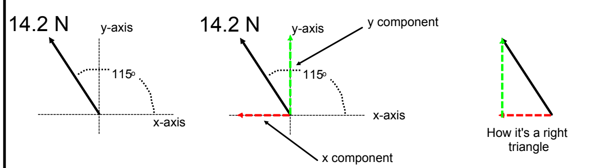
$C_x = C\cos\theta$

$C_y = C\sin\theta$

Important!

Resolving Vectors Example

Decompose the following force vector into its x and y components:



$$C_x = C\cos\theta = 14.2\cos 115^\circ = -6.0\,N$$

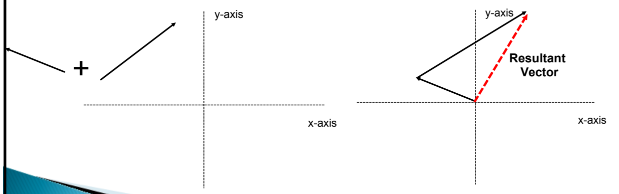
$$C_y = C\sin\theta = 14.2\sin 115^\circ = 12.9\,N$$

Pictorial Vector Addition

Vectors are often shown as arrows drawn to scale, showing magnitude and direction.

Tip to Tail method: Start the first vector at the origin, then draw the second one starting at the tip of the first. Then, draw a line from the origin to the tip of the second vector.

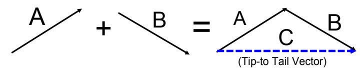
Ex: Add the following vectors:



Adding Vectors: Important!!

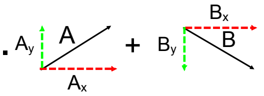
When given one or more vectors to combine:

Ex:



1. decompose all vectors into x and y components,
2. add all x components, and all y components separately,
3. find magnitude using Pythagorean theorem,
4. find direction using tangent relation:

1.



2. $A_x + B_x = C_x$

$A_y + B_y = C_y$

3. $C = \sqrt{C_x^2 + C_y^2}$

4. $\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right)$

Homework 3.1

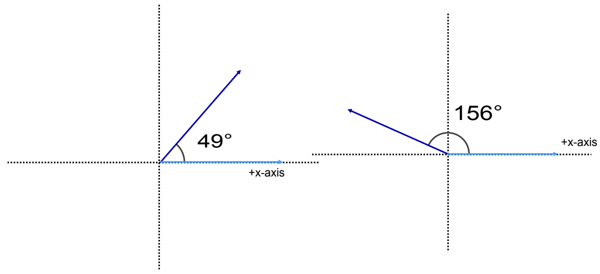
Read 3.3 in your books

3.1 Problems in your Booklets

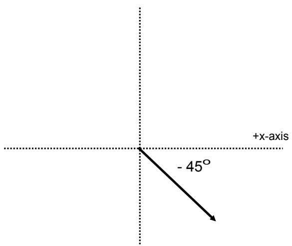
Due: Next Class

Vector Angle Workshop

Calculating angles can result in confusing results in vector problems. Usually a direction angle is expressed as a value measured from the positive x-axis, from 0° to 360°.

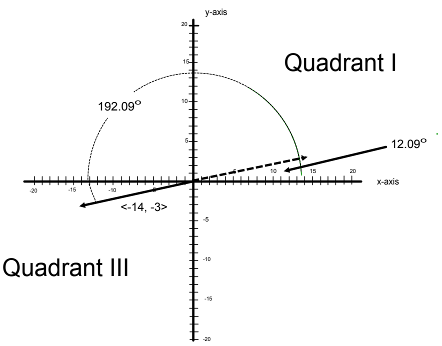


It can also have a negative value, or be said to be "below the positive x-axis."



"45° below the positive x-axis"

In the previous problem, the first result we got on our calculators had to be adjusted to reflect its position in the third quadrant. Our vector components yielded a value of 12.09°, which is in the first quadrant. Since our vector had negative values of both x and y, it is in the third quadrant.

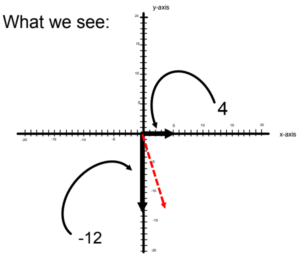


A Calculator's Eye View

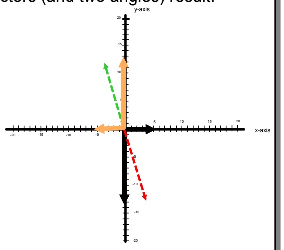
When we see the tangent function that determines angles, we know which quadrant the vector is in by the clearly marked signs.
A calculator's programming makes no distinctions, so it produces results that require interpretation.

Example: $\theta = \tan^{-1}\left(\frac{-12}{4}\right)$

What we see:

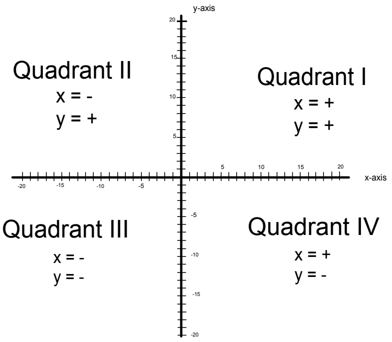


What the calculator sees:
$\theta = \tan^{-1}(-3)$
This could have been made from either <4, -12>, or <-4, 12>, so two possible vectors (and two angles) result.



How to Deal

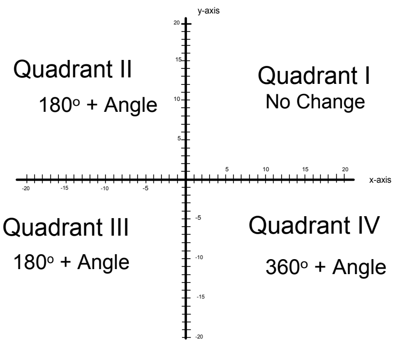
Sometimes you will have to draw your vector to determine where it is. Look at the values of x and y to see which quadrant it is in, and make adjustments.



Adjustments

Assume: $\theta = \tan^{-1}$ (y/x) (where one leg of triangle is on x-axis)

In Quadrants II and III, add 180° to your angle.
Quadrant IV, add 360° to your angle.



Examples:

In partners, sketch and determine the angles of the following vectors.

<-8, -2.7> 198.6°

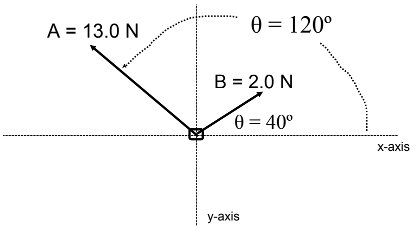
<8, -3> 339.4°

<-12, 14> 130.6°

<4, 6> 56.3°

Example:

Two force vectors (measured in newtons, N), A and B, are pulling on a box. What's the resultant force?



Example:

Decompose:

$A_x = 13 \cos 120° = -6.5$ N $\quad A_y = 13 \sin 120° = 11.3$ N

$B_x = 2 \cos 40° = 1.5$ N $\quad B_y = 2 \sin 40° = 1.3$ N

Add:

x: -6.5 N + 1.5 N = -5.0 N

y: 11.3 N + 1.3 N = 12.6 N

Magnitude:

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{\left(-5.0N^2 + 12.6N^2\right)} = 13.6N = 14N$$

Angle:

$$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{12.6N}{-5.0N}\right) = -68.6^\circ$$

$$\textit{Adjust for 2nd Quadrant}: -68.6^\circ + 180^\circ = 112^0$$