3.1 Vector Manipulation

Trig Review
This is in your AP Test Resources if you need it.

Remember the relations for right triangles:

\[ S = \frac{O}{H}, \quad \sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}} \]
\[ C = \frac{A}{H}, \quad \cos \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} \]
\[ T = \frac{O}{A}, \quad \tan \theta = \frac{\text{Opposite}}{\text{Adjacent}} \]

Read: "The sine of angle theta is equal to the ratio of the opposite leg over the hypotenuse."

Trig Example: BE IN DEGREES!!
Find the missing sides and angles for the following:

Find Hypotenuse:

\[ C = \frac{A}{H}, \quad \cos \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} \]

Find Opposite side:

\[ T = \frac{O}{A}, \quad \tan \theta = \frac{\text{Opposite}}{\text{Adjacent}} \]

Find third angle: the sum of all interior angles of a triangle = 180°.

Vectors

Notation: Represented by an arrow above a variable:

\[ \vec{A} \text{ means } "\text{Vector } A" \]

Vectors have x and y components:

Often written: \(<x, y>\).

Resolving (Decomposing) Vectors

Based on trigonometry relations: a right triangle's hypotenuse is the vector sum of its legs.

Given a vector's magnitude and direction, decompose it into its x and y components:

\[ C_x = C \cos \theta \]
\[ C_y = C \sin \theta \]

Important!

Resolving Vectors Example

Decompose the following force vector into its x and y components:

\[ 14.2 \text{ N} \]

\[ 115° \]

\[ C_x = C \cos \theta = 14.2 \cos 115° = -6.0 \text{ N} \]

\[ C_y = C \sin \theta = 14.2 \sin 115° = 12.9 \text{ N} \]
Pictorial Vector Addition

Vectors are often shown as arrows drawn to scale, showing magnitude and direction.

**Tip to Tail** method: Start the first vector at the origin, then draw the second one starting at the tip of the first. Then, draw a line from the origin to the tip of the second vector.

Ex: Add the following vectors:

\[ \begin{align*}
\vec{A} + \vec{B} &= \vec{C} \\
\vec{A} + \vec{B} &= (A_x, A_y) + (B_x, B_y) \\
\vec{C} &= (C_x, C_y)
\end{align*} \]

Adding Vectors: Important!!

When given one or more vectors to combine:

1. decompose all vectors into x and y components,
2. add all x components, and all y components separately,
3. find magnitude using Pythagorean theorem,
4. find direction using tangent relation:

\[ \begin{align*}
\vec{A} + \vec{B} &= \vec{C} \\
\vec{A} + \vec{B} &= (A_x, A_y) + (B_x, B_y) \\
\vec{C} &= (C_x, C_y)
\end{align*} \]

Homework 3.1

Read 3.3 in your books
3.1 Problems in your Booklets
Due: Next Class

Vector Angle Workshop

Calculating angles can result in confusing results in vector problems. Usually a direction angle is expressed as a value measured from the positive x-axis, from 0° to 360°.

In the previous problem, the first result we got on our calculators had to be adjusted to reflect its position in the third quadrant. Our vector components yielded a value of 12.09°, which is in the first quadrant. Since our vector had negative values of both x and y, it is in the third quadrant.

It can also have a negative value, or be said to be "below the positive x-axis."
A Calculator’s Eye View
When we see the tangent function that determines angles, we know which quadrant the vector is in by the clearly marked signs. A calculator's programming makes no distinctions, so it produces results that require interpretation.

Example: \( 0 = \tan^{-1}\left(\frac{-12}{4}\right) \)

What we see:

What the calculator sees:
This could have been made from either \(-4, -12\), or \(-4, 12\), so two possible vectors (and two angles) result.

Example:

How to Deal
Sometimes you will have to draw your vector to determine where it is. Look at the values of x and y to see which quadrant it is in, and make adjustments.

Adjustments
Assume: \( 0 = \tan^{-1}\left(\frac{y}{x}\right) \) (where one leg of triangle is on x-axis)

In Quadrants II and III, add 180° to your angle.
Quadrant IV, add 360° to your angle.

Examples:
In partners, sketch and determine the angles of the following vectors.

\(<-8, -2.7>\)
\(<-8, -3>\)
\(<-12, 14>\)
\(<4, 6>\)

\(198.6°\)
\(339.4°\)
\(130.6°\)
\(56.3°\)

Example:
Two force vectors (measured in newtons, N), A and B, are pulling on a box. What’s the resultant force?

\(A = 13.0 \text{ N} \quad \theta = 120°\)
\(B = 2.0 \text{ N} \quad \theta = 40°\)

\(x: -6.5 \text{ N} + 1.5 \text{ N} = -5.0 \text{ N} \)
\(y: 11.3 \text{ N} + 1.3 \text{ N} = 12.6 \text{ N} \)

Magnitude:
\(C = \sqrt{C_x^2 + C_y^2} = \sqrt{(-5.0 N^2 + 12.6 N^2)} = 13.6 N = 14 N\)

Angle:
\(\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{12.6 N}{-5.0 N}\right) = -68.6°\)

Adjust for 2nd Quadrant: \(-68.6° + 180° = 112°\)