3.2 - Components of Motion

Components of Motion
Def: the separate x and y parameters of motion: position, velocity, and acceleration.
Time is always the same for x and y.
Decomposing vectors works the same regardless of physical quantity analyzed.

Ex: Velocity:
\[ v_x = v \cos \theta \]
\[ v_y = v \sin \theta \]

Kinematics of Component Motion
For x and y components - use subscripts to show what's what.

Position:
\[ x = x_0 + v_{x0}t + \frac{1}{2}a_xt^2 \]
\[ y = y_0 + v_{y0}t + \frac{1}{2}a_yt^2 \]

Velocity:
\[ v_x = v_{x0} + a_xt \]
\[ v_y = v_{y0} + a_yt \]

Example I - Position
Starting at the origin, a ball rolls at a constant 0.50 m/s velocity at a 37° angle.
Find its (x, y) coordinates after 3.0 seconds.
Strategy: Draw a picture, decompose velocity, then use the position equation (for x and y separately) to find its location. What does constant velocity imply?

Components of Motion
The direction of the overall velocity vector is obtained the same as any other vector too:
\[ \theta = \tan^{-1} \left( \frac{v_y}{v_x} \right) \]
Example 1 - Position

Decompose velocity:

\[ v_x = v \cos \theta = 0.50 \frac{m}{s} \cdot (\cos 37^\circ) = 0.40 \text{ m/s} \]

\[ v_y = v \sin \theta = 0.50 \frac{m}{s} \cdot (\sin 37^\circ) = 0.30 \text{ m/s} \]

Example 1 - Position

Kinematics: use the position equation (acceleration drops out - constant velocity):

\[ x = x_0 + v_x t = 0 + 0.40 \frac{m}{s} \cdot 3.0 \text{ s} = 1.2 \text{ m} \]

\[ y = y_0 + v_y t = 0 + 0.30 \frac{m}{s} \cdot 3.0 \text{ s} = 0.9 \text{ m} \]

Coordinates = (1.2, 0.9)

Curvilinear Motion

An object displays this when it is accelerated in a direction other than 0° or 180°.

Example 2 - Position

A ball rolling along the x axis at 1.50 m/s receives an acceleration of 2.80 m/s² in the positive y direction.

1. What is the position of the ball 3.0 seconds later?

Draw a picture:

\[ \text{Y direction} \]

\[ x = 1.5 \text{ m/s} \]

\[ a = 2.80 \text{ m/s}^2 \]

Example 2 - Position

\[ x = x_0 + v_x t + \frac{1}{2} a t^2 \]

\[ = 0 + 1.50 \frac{m}{s} \cdot 3.0 \text{ s} + \frac{1}{2} \cdot 2.80 \frac{m}{s}^2 \cdot (3.0 \text{ s})^2 = 4.50 \text{ m} \]

\[ y = y_0 + v_y t + \frac{1}{2} a t^2 \]

\[ = 0 + 0 \frac{m}{s} \cdot 3.0 \text{ s} + \frac{1}{2} \cdot 2.80 \frac{m}{s}^2 \cdot (3.0 \text{ s})^2 = 12.6 \text{ m} \]

Example 3 - Velocity

The same ball rolls along the x axis at 1.50 m/s with an acceleration of 2.80 m/s in the +y direction.

What are the velocity & direction of the ball after 3.0 seconds?

Same picture:

\[ \text{Y direction} \]

\[ x = 1.5 \text{ m/s} \]

\[ a = 2.80 \text{ m/s}^2 \]
**Example 3 - Velocity**

Find x and y component velocities, then find the vector sum of velocity:

\[ v_x = v_{x_0} + a_t t = 1.50 \text{ m/s} + 0 \text{ m/s}^2 \times (3.0 \text{ s}) = 1.50 \text{ m/s} \]

\[ v_y = v_{y_0} + a_t t = 0 \text{ m/s} + 2.80 \text{ m/s}^2 \times (3.0 \text{ s}) = 8.40 \text{ m/s} \]

\[ v = \sqrt{v_x^2 + v_y^2} = \sqrt{(1.50 \text{ m/s})^2 + (8.40 \text{ m/s})^2} = 8.53 \text{ m/s} \]

Are we done?

**Example 3 - Direction**

We have to determine the direction of motion:

\[ \theta = \tan^{-1} \left( \frac{v_y}{v_x} \right) = \tan^{-1} \left( \frac{8.40 \text{ m/s}}{1.50 \text{ m/s}} \right) = 79.9^\circ \]

**Homework**

- Read 3.1 & 3.2 in your books
- 3.2 Problems in your Booklets
- Due: Next Class