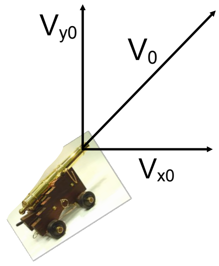
3.3 Projectile Motion – Part 1

Projectile Motion

When a projectile launches, the **muzzle velocity (v_0)** (speed leaving the launcher) can be decomposed:

$$v_{x0} = v_0 \cos\theta$$
$$v_{y0} = v_0 \sin\theta$$

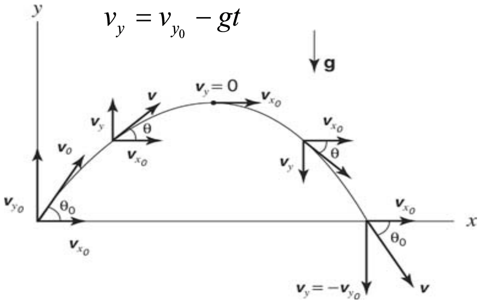


Projectile Motion

Velocity components during flight

$$v_x = v_{x_0}$$ Does not change!

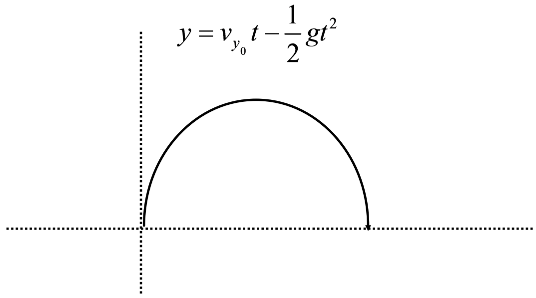
$$v_y = v_{y_0} - gt$$



Projectile Motion

Position (x,y) during flight traces a downwards opening **parabolic** path:

$$x = v_{x_0} t$$
$$y = v_{y_0} t - \frac{1}{2} g t^2$$



Three Projectile Equations (Not in Resources!)

Time to the top of the parabola:

$t_u = \dfrac{v_{y_0}}{g}$	t_u = time up (s) v_{y0} = y velocity (m/s) g = 9.81 m/s^2

Maximum height (m):

$y_{max} = v_{y_0} t_u - \frac{1}{2} g t_u^2$	v_{y0} = y velocity (m/s) t_u = time up (s) g = 9.81 m/s^2

Maximum Range (m)(x position):

$Range = \dfrac{v_0^2 \sin(2\theta)}{g}$	v_0 = muzzle velocity (m/s) θ = launch angle (degrees) g = 9.81 m/s^2

Ranger Details

Note: given some launch velocity, two angles give the same range: the given angle, and that angle subtracted from 90 degrees.

A projectile's maximum range (in a vacuum) is at 45 degrees.

With air resistance, the best angle is a bit less. Can you guess why?

At a lower angle there's a slightly greater initial horizontal velocity, so when projectiles launched at 45 degrees are landing, those at lower angle have already travelled farther.

Example 1

A golf ball is hit with an initial velocity (v_0) of 30.0 m/s at 35°.

How far will it go (what is its range)?

Example 1: Answer

How far will the ball go?

$$Range = \frac{v_0^2 \sin 2\theta}{g} = \frac{(30.0\,m/s)^2 \bullet (\sin(2 \bullet 35^\circ))}{9.81\,m/s^2} = 86.2\,m$$

Follow up question: What other angle could make the ball go that far?

$$90^\circ - 35^\circ = 55^\circ$$

Check it with the range equation.

$$Range = \frac{v_0^2 \sin 2\theta}{g} = \frac{(30.0\,m/s)^2 \bullet (\sin(2 \bullet 55^\circ))}{9.81\,m/s^2} = 86.2\,m$$

Example 2

The same golf ball is hit with an initial velocity (v_0) of 30.0 m/s at 35°.

What is the maximum height?

What do you need to find first? Second?

Example 2: Answer

To find maximum height, we need time to the top.

To find that, we need the initial vertical velocity component, v_{y0}

$$v_{y_0} = v_0 \sin\theta = 30.0\,m/s \bullet \sin 35^\circ = 17.2\,m/s$$

Example 2: Answer

Time to the top:

$$t_u = \frac{v_{y_0}}{g} = \frac{17.2\,m/s}{9.81\,m/s^2} = 1.75\,s$$

Max height:

$$y_{max} = v_{y_0} t_u - \frac{1}{2} g t_u^2$$

$$= 17.2\,m/s \bullet 1.75\,s - \frac{1}{2} \bullet 9.81\,m/s^2 \bullet (1.75\,s)^2 = 15.1\,m$$

Homework 3.3

3.3 Problems in your Booklets
Due: Next Class

Vectors Quiz Tomorrow!