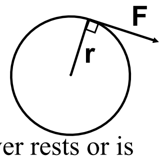
9.2 Torque, Equilibrium, and Stability

Torque (τ)

A force causing rotational motion is torque; Latin: means "to twist."

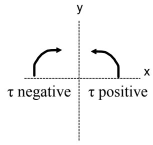
Torque depends on:
1. force applied.
2. radial distance from axis to force.
3. force angle.



Fulcrum: the pivot point on which a lever rests or is supported.

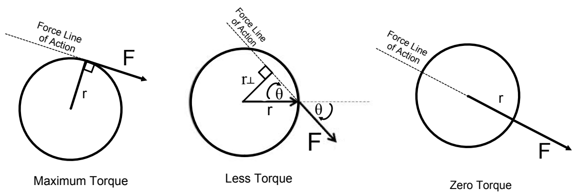
Torque is a vector: Right Hand Rule.

Negative torque (-) causes clockwise motion; counterclockwise is positive (+).



Direction of Force

The closer the force line of action is to perpendicular, the more torque.

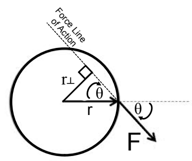


Ex: push a heavy door close to the hinge and little happens; at the farthest edge it moves easier.

Torque Math

Torque equation:

$\tau = r_{\perp} F$	τ = torque (m•N)
$= rF \sin\theta$	r = radius (m)
AP Equations	F = Force (N)
	θ = angle (degrees)

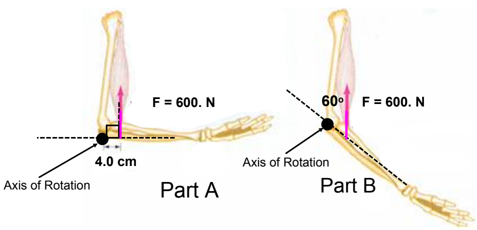


Units are meter • newtons (m • N): same as that of work (N • m = J), but torque is **NOT** a measure of energy!

Energy comes later.

Arm Example

Your arm moves when muscles connected to your elbow contract. What torque is generated in situations A and B, if the radius is 4.0 cm, and the force produced by the biceps is 600. N?

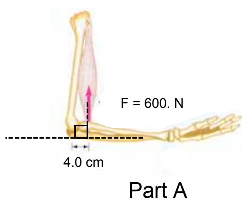


Arm Answer (Part A)

The line of force is perpendicular to the radius through the axis of rotation, so:

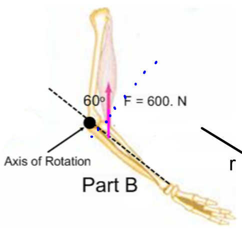
$$\tau = r \bullet F \bullet \sin\theta$$

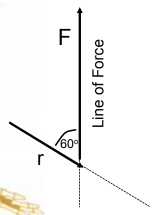
$$= 0.04\,m \bullet 600.\,N \bullet \sin 90 = 24\,m \bullet N$$

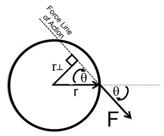


Arm Answer (Part B)

The line of force is not perpendicular to the radius through the axis of rotation: use the lever angle with respect to the direction of force.



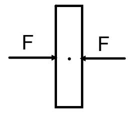




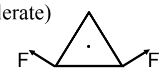
$$\tau = r \bullet F \bullet \sin\theta$$
$$= 0.04\,m \bullet 600.\,N \bullet \sin 60$$
$$= 21\,m \bullet N$$

Translational vs. Rotational Equilibrium

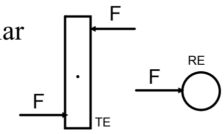
Balanced linear forces yield translational equilibrium (TE) (no acceleration; object could move).



Balanced torque yields rotational equilibrium (RE).
(Object can rotate, NOT accelerate)



An object in TE could have angular acceleration, an object in RE could have linear acceleration:



Mechanical vs. Static Equilibrium

When TE and RE are present, an object is in mechanical equilibrium (ME).

Objects in ME can either be at rest or moving with a constant linear and/or angular velocity.

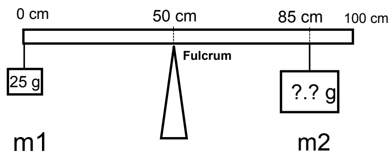
Static equilibrium: an object totally at rest.

This is why you shovel your shed!

Equilibrium Problem

Two masses hang from the meter stick as shown in the picture. How much would m2 have to be for the stick to remain in equilibrium?

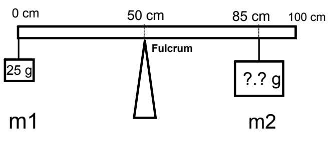
Note: distances should from the fulcrum.



Equilibrium Answer

Gravity pulls on the masses, producing torque.

Compute torque of m1:



$25\,g\ mass: \tau = r_\perp \bullet F$

$$= 0.5\,m \bullet 0.025\,kg \bullet 9.81\,m/s^2 = 0.123\,m \bullet N$$

Set net torque (zero, in equilibrium) equal to the torque equation and solve to find mass:

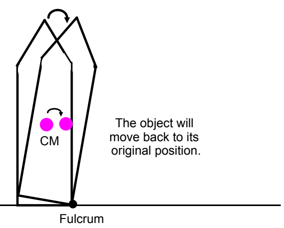
$$\tau = r_\perp \bullet F = r_\perp \bullet m \bullet g$$

$$m = \frac{\tau}{r_\perp \bullet g} = \frac{0.123\,m \bullet N}{0.35\,m \bullet 9.81\,m/s^2} = 0.036\,kg$$

Stability and Center of Mass

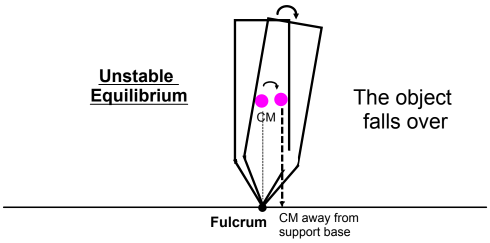
Stable equilibrium: small displacement doesn't tip object - CM is over base.

Stable Equilibrium



Unstable Equilibrium

Unstable equilibrium, small displacement knocks an object over.

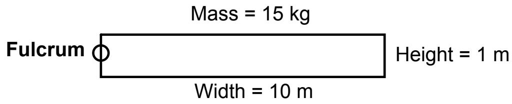


Center of Mass Note

Sometimes a symmetric object (of uniform composition) is tipped, or acts as a lever.

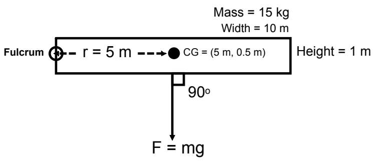
For calculations, consider CM to be in the middle.

Example: what torque does this rectangle experience?



CM Answer

The CM can be considered to be exactly in the middle of the rectrangle (5 m, 0.5 m), so torque is as follows:



$$\tau = r \bullet F \bullet \sin\theta = r \bullet m \bullet g \bullet \sin\theta$$
$$= 5\,m \bullet 15\,kg \bullet 9.81\,m/s^2 \bullet \sin 90^\circ$$
$$= 740\,m \bullet N$$

Homework 9.2

Read 8.3 in your books
Problems 9.2 in your Booklet
Due: Next Class