9.2 Torque, Equilibrium, and Stability

Torque (\(\tau\))
A force causing rotational motion is torque. Latin: means "to twist."
Torque depends on:
1. force applied.
2. radial distance from axis to force.
3. force angle.
Fulcrum: the pivot point on which a lever rests or is supported.
Torque is a vector: Right Hand Rule.
Negative torque \((\tau)\) causes clockwise motion; counterclockwise is positive \((\tau)\).

Direction of Force
The closer the force line of action is to perpendicular, the more torque.
Ex: push a heavy door close to the hinge and little happens; at the farthest edge it moves easier.

Torque Math
Torque equation:
\[
\tau = rF \sin \theta
\]
Units are meter \(\times\) newtons \((m \times N)\): same as that of work \((N \times m = J)\), but torque is NOT a measure of energy!
Energy comes later.

Arm Example
Your arm moves when muscles connected to your elbow contract. What torque is generated in situations A and B, if the radius is 4.0 cm, and the force produced by the biceps is 600. N?

Arm Answer (Part A)
The line of force is perpendicular to the radius through the axis of rotation, so:
\[
\tau = rF \sin \theta
\]
\[
= 0.04 m \times 600. N \times \sin 90 = 24 m \times N
\]
**Arm Answer (Part B)**
The line of force is not perpendicular to the radius through the axis of rotation: use the lever angle with respect to the direction of force.

\[ \tau = r \cdot F \cdot \sin \theta \\
= 0.04 \cdot 600 \cdot N \cdot \sin 60 \\
= 21m \cdot N \]

**Translational vs. Rotational Equilibrium**
Balanced linear forces yield translational equilibrium (TE) (no acceleration; object could move).

Balanced torque yields rotational equilibrium (RE). (Object can rotate, NOT accelerate)

An object in TE could have angular acceleration, an object in RE could have linear acceleration:

**Mechanical vs. Static Equilibrium**
When TE and RE are present, an object is in mechanical equilibrium (ME).

Objects in ME can either be at rest or moving with a constant linear and/or angular velocity.

Static equilibrium: an object totally at rest.

This is why you shovel your shed!

**Equilibrium Problem**
Two masses hang from the meter stick as shown in the picture. How much would m2 have to be for the stick to remain in equilibrium? Note: distances should from the fulcrum.

**Equilibrium Answer**
Gravity pulls on the masses, producing torque.

Compute torque of m1:

\[ \tau = r \cdot g \cdot m = 0.35 \cdot 9.8 \cdot m \cdot N = 0.123 \cdot m \cdot N \]

Set net torque (zero, in equilibrium) equal to the torque equation and solve to find mass:

\[ m = \frac{\tau}{r \cdot g} = \frac{0.123 \cdot m \cdot N}{0.35 \cdot 9.8 \cdot m/s^2} = 0.036 \, \text{kg} \]

**Stability and Center of Mass**
Stable equilibrium: small displacement doesn't tip object - CM is over base.
**Unstable Equilibrium**

Unstable equilibrium, small displacement knocks an object over.

The object falls over.

**CM Answer**

The CM can be considered to be exactly in the middle of the rectangle (5 m, 0.5 m), so torque is as follows:

\[ \tau = r \cdot F \cdot \sin \theta = r \cdot m \cdot g \cdot \sin \theta \]

\[ = 5m \cdot 15kg \cdot 9.81 m/s^2 \cdot \sin 90^\circ \]

\[ = 740m \cdot N \]

**Center of Mass Note**

Sometimes a symmetric object (of uniform composition) is tipped, or acts as a lever.

For calculations, consider CM to be in the middle.

Example: what torque does this rectangle experience?

**Homework 9.2**

Read 8.3 in your books

Problems 9.2 in your Booklet

Due: Next Class