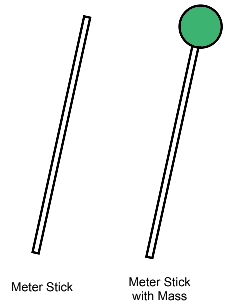
9.3 Rotational Dynamics

Physics Democracy!

Which hits first? Vote! - Answer Later

Mass	No Mass	Same Time



Review

Recall: angular acceleration is the rate of change of rotational speed:

$\alpha = \frac{\Delta\omega}{t}$	α = angular acceleration (rad/s^2) ω = angular speed (rad/s) t = time (s)

We need this relation as we get deeper into torque.

Moment of Inertia (MoI)

When we applied F = ma to objects, we considered mass concentrated at a point: motion was linear.

With rotations, we use **moment of inertia (MoI)** (Symbol: I) to replace mass.

Consider any object under torque - all parts respond differently depending on location.

Moment of inertia sums individual responses to torque.

F = ma : Torque Analogue

Torque is calculated:

$\tau_{net} = I\alpha$ AP Equation	I = moment of inertia (kg•m^2) (Resource P. 5) α = angular acceleration (rad/s^2)

Note: AP Resources show this relation thusly: $\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$

MoI measures rotational inertia: a body's tendency to resist rotational change (mass resists linear motion changes).

MoI depends on the mass distribution of a body relative to its rotational axis.

Sphere Example 1

A uniform 12.0 kg solid sphere (radius = 0.20 m) has a 13 m•N torque applied to it.

1. What is the sphere's moment of inertia?

Hint: use the table on Resources 5.

$$I_{Solid\ Sphere} = \frac{2mr^2}{5} = \frac{2 \bullet 12kg \bullet (0.20m)^2}{5} = 0.192\, kg \bullet m^2$$

Sphere Example 2

A uniform 12.0 kg solid sphere (radius = 0.20 m) has a 13 m•N torque applied to it.

2. What angular acceleration does it have?

MoI was 0.192 kg•m².

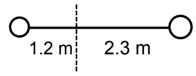
$$\tau_{net} = I\alpha$$

$$\alpha = \frac{\tau}{I} = \frac{13\,m \bullet N}{0.192\,kg \bullet m^2} = 68\ rad/s^2$$

Two Point Masses Ex. 1

Two point masses of 0.15 kg each are positioned at 1.2 m, and 2.3 m from an axis.

What is the moment of inertia of this two point system? (Hint: MoI is additive)



$$I = mr^2$$
$$= 0.15\,kg \bullet (1.2\,m)^2 = 0.216\,kg \bullet m^2$$

$$I = mr^2$$
$$= 0.15\,kg \bullet (2.3\,m)^2 = 0.794\,kg \bullet m^2$$

+

Total = 1.01 kg•m²

Two Point Masses Ex. 2

Two point masses of 150 grams each are positioned at 1.2 m, and 2.3 m from an axis.

If the two points are connected by a massless rod of 3.5 m, what will be their MoI? (Hint: this alters the axis of rotation to the CM of the system: Use MoI of two point mass system (Resources 5).)

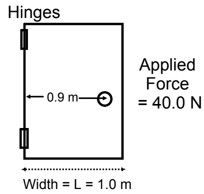
$$I = \frac{Mm}{M+m}x^2$$
$$= \frac{(0.15\,kg)^2}{0.15\,kg + 0.15\,kg}(3.5\,m)^2 = 0.92\,kg \bullet m^2$$

Pushing a Door Ex. 1

A 1.0 m wide, 12.0 kg door opened by applying a constant force of 40.0 N a perpendicular distance 0.90 m away from the hinges.

What is the MoI of the door?

MoI of rectangular solid:



$$I = \frac{mr^2}{3} = \frac{12\,kg \bullet (1.0\,m)^2}{3} = 4.0\,kg \bullet m^2$$

Pushing a Door Ex. 2

A 1.0 m wide, 12.0 kg door is opened by applying a constant force of 40.0 N at a perpendicular distance 0.90 m away from the hinges.

What is its angular acceleration? (Height doesn't matter.)

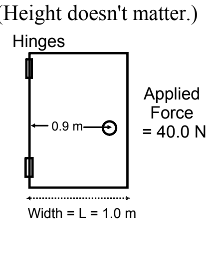
First: net torque:

$$\tau_{net} = r_{\perp}F$$
$$= 0.90\,m \bullet 40.0\,N = 36\,m \bullet N$$

Then acceleration:

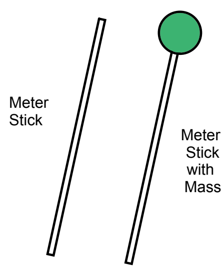
$$\tau_{net} = I\alpha$$

$$\alpha = \frac{\tau_{net}}{I} = \frac{36\,m \bullet N}{4.0\,kg \bullet m^2} = 9.0\,rad/s^2$$



Meter Stick Demo. Answer

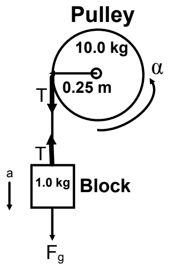
The meter stick with the mass had a greater MoI (approximated by a point mass: I = mr²), and resisted rotation more.



Pulley Problem Ex. 1

A 1.0 kg block hangs from a string wrapped around a 10.0 kg frictionless solid disk pulley with a 0.25 m radius.

What is the MoI of the pulley?



Solid disk's MoI:

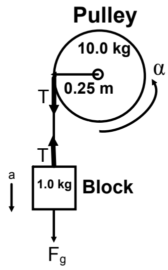
$$I = \frac{1}{2}mr^2$$

$$= \frac{1}{2}10.0\,kg \bullet (0.25\,m)^2 = 0.31\,kg \bullet m^2$$

Pulley Problem Ex. 2

A 1.0 kg block hangs from a string wrapped around a 10.0 kg frictionless solid disk pulley with a 0.25 m radius and MoI of 0.31 kg•m².

If the block descends under the influence of gravity, what torque does it have?



$$\tau = r_\perp F = r_\perp \bullet m \bullet g$$

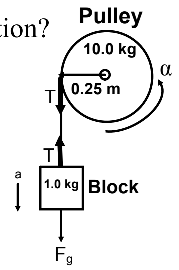
$$= 0.25\,m \bullet 1.0\,kg \bullet 9.81\,m/s^2$$

$$= 2.45\,m \bullet N$$

Pulley Problem Ex. 3

A 1.0 kg block hangs from a string wrapped around a 10.0 kg frictionless solid disk pulley with a 0.25 m radius and MoI of 0.31 kg•m².

What's the pulley's angular acceleration?



$$\tau = I\alpha$$

$$\alpha = \frac{\tau}{I} = \frac{2.45\,m \bullet N}{0.31\,kg \bullet m^2} = 7.85\,rad/s^2$$

Homework 9.3

Read 8.4 in your book

9.3 Booklet Problems

Due: Next Class