9.4 Rotational Kinematics

Rotational Kinematics

You dealt with kinematics: the study of motion.

Making these substitutions results in rotational kinematics:

Linear Term		Rotational Analogue	
Position	x (m)	Angle	θ (rad)
Velocity	v (m/s)	Angular Velocity	ω (rad/s)
Acceleration	a (m/s²)	Angular Acceleration	α (rad/s²)

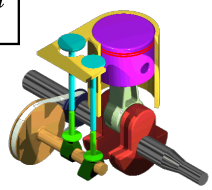
Rotational Equations

Three equations:

Angular Position: $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$ AP Equation

Angular Speed: $\omega = \omega_0 + \alpha t$ AP Equation

Sandbag: $\omega^2 = \omega_0^2 + 2\alpha(\Delta\theta)$



Press the Button! →

Lots of Circular Motion!

Bike Wheel Example

A bike is elevated so its front wheel freely rotates. A kid turns it (from rest) for 15 seconds with his hand until it reaches 49 rpm.

What angular acceleration did the wheel undergo?

First, convert ω: rpm to rad/s:

$$49\,rpm \bullet \frac{1\,\text{min}}{60\,s} \bullet \frac{2\pi\,rad}{rotation} = 5.131\,rad/s$$

$$\omega = \omega_0 + \alpha t$$

$$\alpha = \frac{\omega}{t} = \frac{5.131\,rad/s}{15\,s} = 0.34\,rad/s^2$$

Bike Example Part 2

How many rotations did the bike wheel undergo as it was being turned?

Pertinent values from previous problem:

$\Delta\omega = 49$ rpm
$\omega_0 = 0.0$ rpm
$t = 15$ s
$\alpha = 0.34$ rad/s²

Find θ, then convert to rotations.

Bike Example Part 2 Answer

Using angular position equation:

$\Delta\omega = 49$ rpm
$\omega_0 = 0.0$ rpm
$t = 15$ s
$\alpha = 0.34$ rad/s²

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

Terms Drops Out

$$= \frac{1}{2}0.34\,rad/s^2(15\,s)^2$$

$$= 38.25\,rad$$

Finally, convert to rotations:

$$38.25\,rad \bullet \frac{1\,rotation}{2\pi\,rad} = 6.1\,rotations$$

Involved Example

A force of 14 N is applied perpendicularly to the edge of a stationary flywheel (r = 0.45 m). If the MoI of the flywheel is 35 kg m^2, how fast will it be rotating (in rpm) after 230 rotations?

Information:
F = 14 N
r = 0.45 m
I = 35 kg m^2
θ = 230 rotations (convert to rad)

$$230\,rotations \bullet \frac{2\pi\,rad}{rotation} = 1444.4\,rad$$

Involved Example Answer 1

First, calculate the torque:

$$\tau = F \bullet r_{\perp}$$
$$= 14\,N \bullet 0.45\,m = 6.3\,m \bullet N$$

use that to find angular acceleration:

$$\tau = I \bullet \alpha$$
$$\alpha = \frac{\tau}{I} = \frac{6.3\,m \bullet N}{35\,kg \bullet m^2} = 0.18\,rad/s^2$$

Involved Example Answer 2

Finally, use the sandbag equation:

$$\omega^2 = \omega_0^2 + 2\alpha(\Delta\theta)$$
$$\omega = \sqrt{\omega_0^2 + 2\alpha(\Delta\theta)}$$
$$\omega = \sqrt{0 + 2 \bullet 0.18\,rad/s^2 \bullet 1444.4\,rad} = 23\,rad/s$$

and convert to rpm:

$$22.80\,rad/s \bullet \frac{60\,s}{m} \bullet \frac{1\,rotation}{2\pi\,rad} = 220\,rpm$$

Homework 9.4

Read 7.5 in your book
Problems 9.4 in your Booklet
Due: Next Class