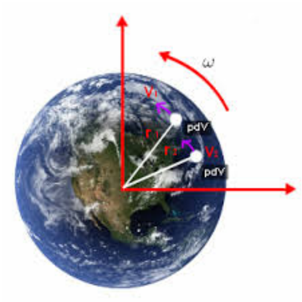
9.5 Rotational Work and Rotational Energy



Rotational Work

Analogous to linear work, but force is replaced by torque; distance replaced by angular displacement:

$W = Fs$	W = work (J) F = linear force (N) s = arc length (m)
$W = F(r_{\perp} \cdot \theta)$	r⊥ = radius (m) θ = angle (rad)
$W = \tau \cdot \theta$	τ = torque (m•N)
$W = I \cdot \alpha \cdot \theta$	I = moment of inertia (kg•m²) α = angular acceleration (rad/s²)

Rotational Power

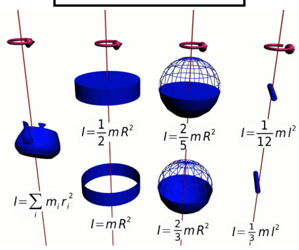
Is derived directly from rotational work, in that it is an amount of energy discharged in an amount of time:

$P = \frac{\Delta E}{\Delta t}$ AP Equation	$= \tau\left(\frac{\theta}{t}\right) = \tau\omega$	P = Power (J/s) E = Energy (J) t = time (s)
		τ = torque (m•N) θ = angle (rad)
		ω = angular speed (rad/s)

Rotational Kinetic Energy

For any rotating body:

$$K_{Rotational} = \frac{1}{2} I \omega^2$$



Rotational Work-Energy Theorem

We learned that kinetic energy equalsan amount of work done:

$$W_{net} = K - K_0 = \Delta K$$

In rotational motion, this can be derived thusly:

$$W_{net} = I\alpha\theta$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\alpha\theta = \frac{\omega^2 - \omega_0^2}{2}$$

$$W_{net} = I\left(\frac{\omega^2 - \omega_0^2}{2}\right) = \frac{1}{2}I\omega^2 - \frac{1}{2}I\omega_0^2 = \Delta K$$

Rotating, Translating Bodies

What if you've got a rotating, AND translating body, like a rolling wheel (no slipping)?

Look at rotational **and** translational dynamics, using center of mass (CM):

Rotational: $K_{rotational} = \frac{1}{2} I_{CM} \omega^2$

Translational: $K_{translational} = \frac{1}{2} m v_{CM}^2$

Combined: $K_{total} = \frac{1}{2} I_{CM} \omega^2 + \frac{1}{2} m v_{CM}^2$

Rotational Energy Problem 1

The CM of a solid 1.0 kg cylinder (I = 1/2 mr²) rolls without slipping at a speed of 1.8 m/s.

What is the cylinder's rotational kinetic energy?

Note 1: radius cancels out.

Note 2: review equation:

$$\omega = \frac{v_{CM}}{r}$$

$$K_{rotational} = \frac{1}{2} I \omega^2 = \frac{1}{2}\left(\frac{1}{2} mr^2\right)\left(\frac{v_{CM}}{r}\right)^2$$

$$= \frac{1}{4} m v_{CM}^2 = \frac{1}{4}(1.0\,kg)(1.8\,m/s)^2 = 0.81\,J$$

Rotational Energy Problem 2

A uniform, solid 1.0 kg cylinder (I = 1/2 mr²) rolls without slipping at a speed of 1.8 m/s.

What is the cylinder's translational kinetic energy?

$$K_{translational} = \frac{1}{2} m v_{CM}^2 = \frac{1}{2}(1.0\,kg)(1.8\,m/s)^2 = 1.6\,J$$

$$K_{total} = \frac{1}{2} I_{CM} \omega^2 + \frac{1}{2} m v_{CM}^2$$

$$= 0.81\,J + 1.62\,J = 2.4\,J$$

Rotational Energy Problem 3

A uniform, solid 1.0 kg cylinder (I = 1/2 mr²) rolls without slipping at a speed of 1.8 m/s.

What percentage of the total is rotational kinetic energy?

$$\frac{K_{Rotational}}{K_{Total}} = \frac{0.81\,J}{2.4\,J} = 33.8\%$$

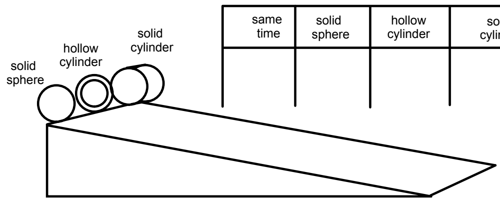
Thus, most of the energy is in translational form.

Inertial Race Physics Democracy!

Consider a sphere, a solid cylinder, and a hollow cylinder (like an empty can) of the same masses and radii. Which will reach the bottom first, if you roll them down an incline?

Vote!

same time	solid sphere	hollow cylinder	solid cylinder



https://www.youtube.com/watch?v=tcs93mPn91E

Inertial Race Answer

The sphere wins every time, its moment of inertia is the lowest, so its angular acceleration is largest:

$$\alpha = \frac{\tau}{I}$$

Sphere: $I = \frac{2mr^2}{5}$

Solid Cylinder: $I = \frac{1}{2} mr^2$

Hollow Cylinder: $I = mr^2$

Homework 9.5

Read 8.5 in your book

Problems 9.5 in your Booklet

Due: Next Class