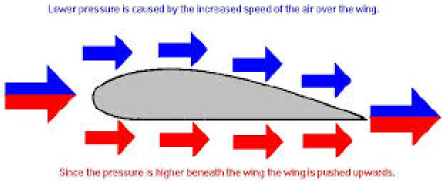
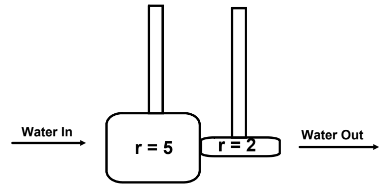
1.5 Bernoulli's Principle



1. Physics Democracy!

The contraption consists of two sections of pipe with different radii. The radial comparisons are listed. Which pipe will have the tallest column of water? Which the shortest?



Tallest:	R = 5	R = 2	There Will Be No Difference

Bernoulli's Principle

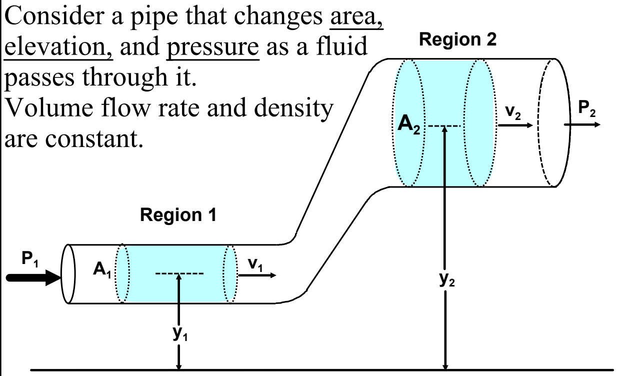
In 1738, Swiss mathematician Daniel Bernoulli derived an equation relating pressure, fluid velocity, and height.

It uses the work-energy theorem: net work done by a fluid equals its change in kinetic energy (via velocity) and potential energy (via height):

$W_{net} = \Delta K + \Delta U$	W = Work (J) K = Kinetic Energy (J) U = Potential Energy (J)

Fluid Changes

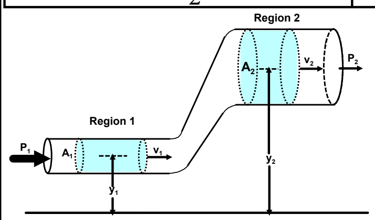
Consider a pipe that changes area, elevation, and pressure as a fluid passes through it.
Volume flow rate and density are constant.



Bernoulli's Equation (AP # 6)

For a fluid that could change velocity, elevation, and pressure between regions 1 and 2:

$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$	P = pressure (Pa:N/m^2) ρ = density (kg/m^3) v = velocity (m/s) g = 9.81 m/s^2 y = height (m)

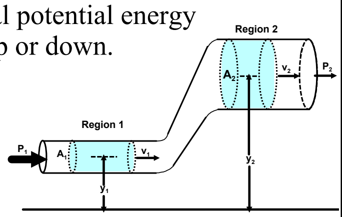


Three Parameters Explained

Pressure (P): Caused by system: a pump, piston, balloon under tension, etc. pushing against the fluid. ALSO: If either region is open to the atmosphere, that pressure = 1.0 E 5 Pa.

Velocity (v): Driven by area changes. Use Pascal's Principle to determine velocity ($A_1v_1=A_2v_2$).

Height (y): Gravitational potential energy changes as water goes up or down.



Unit Analysis

Note: all three terms of Bernoulli's Equation have interesting base units. As a skill test derive each one:

P = pressure (Pa = N/m^2)
ρ = density (kg/m^3)
v = velocity (m/s)
g = 9.81 m/s^2
y = height (m)
N = Force: kg m/s^2

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1$$

$$\frac{N}{m^2} = \frac{\frac{kg \bullet m}{s^2}}{m^2} = \boxed{\frac{kg}{m \bullet s^2}} \qquad \frac{kg}{m^3} \bullet \frac{m^2}{s^2} = \boxed{\frac{kg}{m \bullet s^2}} \qquad \frac{kg}{m^3} \bullet \frac{m}{s^2} \bullet m = \boxed{\frac{kg}{m \bullet s^2}}$$

Just what is a kilogram per meter second squared? How could this be useful?

Unit Breakthrough!

The odd units of the previous slide have meaning!

Bernoulli's equation is an **energy analysis** (stored energy of pressure, moving fluid's kinetic energy, potential energy of elevated fluid).

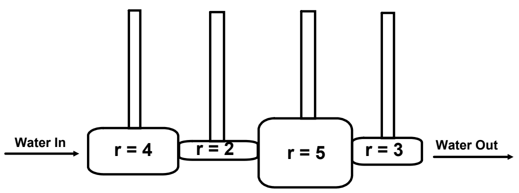
Remembering the Joule's definition: $1J = \frac{1 kg \bullet m^2}{s^2}$

Multiplying the odd conglomeration of base units by m^2/m^2 (which equals 1), you get a value of energy per volume:

$$\frac{kg}{m \bullet s^2} \bullet \frac{m^2}{m^2} = \frac{kg \bullet m^2}{m^3 s^2} = \frac{kg \bullet m^2}{s^2} \bullet \frac{1}{m^3} = \frac{J}{m^3}$$

2. Physics Democracy! FIX!!

The contraption consists of four sections of pipe with different radii. The radial comparisons are listed. Which pipe will have the tallest column of water? Which the shortest?



Tallest					There Will Be No Difference
Shortest					

Applications: Horizontal Flow

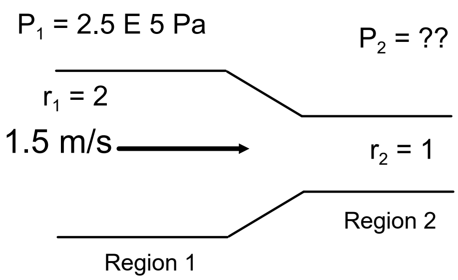
If there is horizontal flow ($y_1 = y_2$), then:

$$\boxed{P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2}$$

This indicates that pressure increases as velocity decreases (and vice versa).

3. Bernoulli Example

Horizontally flowing water is constricted to half the radius. If speed is 1.5 m/s in the larger part (P = 2.5 E 5 Pa), what's the pressure in the smaller part?



3. Example Answer

Using Pascal's Principle, find the velocity in the narrow part, then Bernoulli's to find pressure change:

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{A_1 v_1}{A_2} = \frac{\pi \bullet r_1^2 \bullet v_1}{\pi \bullet r_2^2} = \frac{(2 units)^2 \bullet 1.5 m/s}{(1 unit)^2} = 6.0 m/s$$

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_2 = P_1 + \frac{1}{2}\rho v_1^2 - \frac{1}{2}\rho v_2^2$$

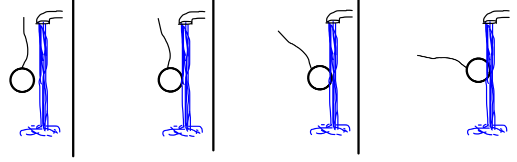
$$= 2.5 E 5 Pa + \frac{1}{2} \bullet 1000 kg/m^3 \left((1.5 m/s)^2 - (6.0 m/s)^2\right)$$

$$\boxed{P_2 = 2.3 E 5 Pa}$$

4. Physics Democracy!

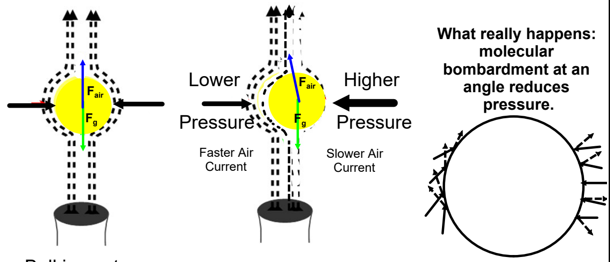
I intend to put the ping pong ball on a string into a running water stream, then move the string away from the water, pulling the ball with it. At what angle will I have to hold the string to get the ball out of the water?

Very little angle: ball separates immediately.	Low angle: ball separates with some effort.	High angle: ball separates with large effort.	Extreme angle: ball will not separate.



Bernoulli's Principle: Ping Pong Ball

As a ping pong ball hovers in an air stream, it stays there due to pressure differentials (demo).

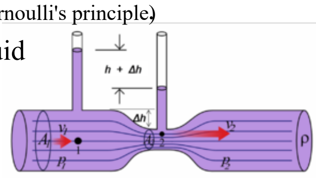


The Venturi Effect

Bernoulli's principle is used in smokestacks: they are built tall so wind may be greater at their tops. Wind blowing over them draws smoke out efficiently.

The Venturi Effect is the reduction in fluid pressure resulting when fluid flows through a constricted section of pipe (based on Bernoulli's principle)

Some meters measure fluid flow based on pressure differentials through a narrow constriction.



Airplane Wings: The Misconception

The 'Bernoulli effect' gives a simplistic explanation for the lift of an airplane.

When air travels over an airplane wing (airfoil), it travels with greater velocity than underneath it.

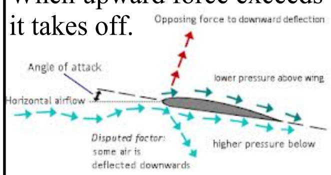
Thus, there is lower pressure above the wing and it rises.

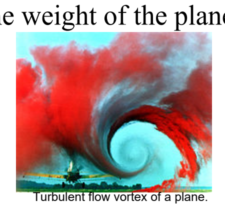
Airplane Wings Continued

In reality, Bernoulli's principle requires ideal fluid flow and energy conservation within the system.

Newton's laws explain this phenomenon better: air striking the wing's bottom is deflected downward, and the reaction to this is the wing's upward motion.

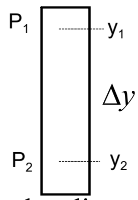
When upward force exceeds the weight of the plane, it takes off.




Turbulent flow vortex of a plane.

Applications: Static Fluid

For a resting fluid, $v_2 = v_1 = 0$, the kinetic energy term of Bernoulli's drops out:



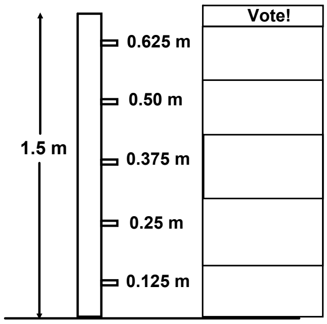
$$P_1 + \rho g y_1 = P_2 + \rho g y_2$$
$$P_1 - P_2 = \rho g y_2 - \rho g y_1$$
$$P_1 - P_2 = \rho g \Delta y$$

This is the pressure-depth relation derived earlier, but height was h, not y. $P = \rho g h$

5. Physics Democracy: Holey Pipes, Batman!

One application of Bernoulli's Equation is finding fluid speed out of a perforated pipe.

Which hole sends water out farthest, (as measured when it reaches table level)?
Why did you vote that way?



Fluid Velocity Derivation

Using Bernoulli's Equation, realize:

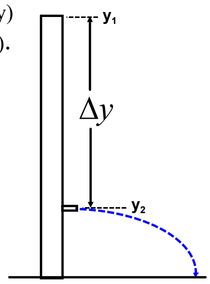
1. P_1 and P_2 are equal (open to atmosphere).
2. $v_1 = 0$ at top of column (zero kinetic energy).
3. $y_2 = 0$ at hole (zero potential energy)

(Note: water may keep falling after exiting).

$$\cancel{P_1} + \cancel{\frac{1}{2}\rho v_1^2} + \rho g y_1 = \cancel{P_2} + \frac{1}{2}\rho v_2^2 + \cancel{\rho g y_2}$$

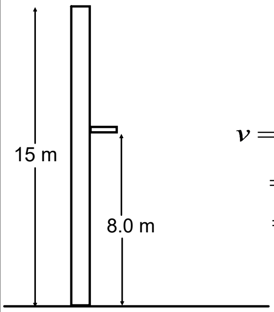
$$\rho g \Delta y = \frac{1}{2}\rho v_2^2$$

$$\boxed{v_2 = \sqrt{2g\Delta y}}$$



6. Water Example

How fast will the water rushing out of a 15 m pipe be if a hole opens up 8 m above the ground?



$$v = \sqrt{2g\Delta y}$$
$$= \sqrt{2 \cdot 9.81\, m/s^2 \cdot 7.0\, m}$$
$$= 11.7\, m/s$$

Homework 1.5

Problems 1.5 in your Booklet
Due: Next Class

Finish Unit 1 Review: Scanned Thursday: 9/6