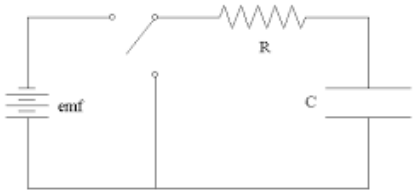
3.C.5 - Resistor/Capacitor (RC) Circuits



Quick Capacitor Review

Capacitors store charge and electrical energy:

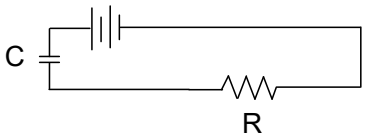
$U_C = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}\bullet\frac{Q^2}{C}$ AP Equations	Non-AP Equation	U_c = Energy (J) Q = Charge (C) V = Voltage (Volts: J/C) C = Capacitance (Farads: C/V)

Resistor/Capacitor (RC) Circuits

Note: RC is NOT 'remote control'.

A battery charges a capacitor, and a serial resistor moderates current.

The larger the resistor, the longer it takes to charge the capacitor.



Time Constant = τ

It takes an infinite time for full charge: charges accumulating on capacitor repel additional charges.

BUT: a usable charge takes a short time.

For calculations, a time constant (τ) of charging is:

$\tau = RC$	R = Resistance (Ohms) C = Capacitance (Farads)

Units are in seconds.

Unit Analysis:

$$R\bullet C = \frac{V}{I}\bullet\frac{C}{V} = \frac{C}{I} = \frac{C}{\frac{C}{s}} = s$$

Charging Capacitors: Voltage

During charging, voltage increases logarithmically.

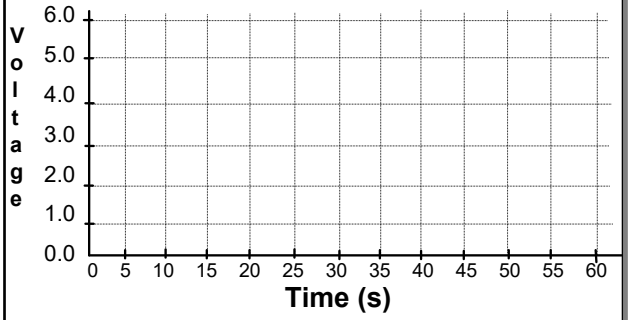
Eventually, capacitor and battery voltage equalize; until then, capacitor voltage is:

$V_C = V_0[1 - e^{-t/\tau}]$	V_C = Capacitor Voltage V_0 = Battery Voltage t = time (s) τ = time constant (s)

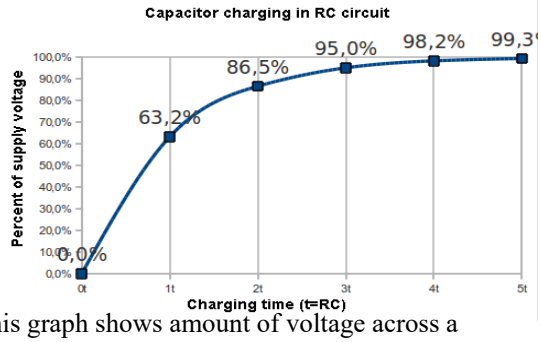
Note: *e* is the base of *natural logarithms*.
It approximately equals 2.718; is common in science.

1. Capacitor Charging Demo.

I will charge the capacitor, and shout out numbers every five seconds. Please write them for a graph.



Charging Capacitors: Voltage Graph



This graph shows amount of voltage across a capacitor at different time constants.

Manipulating Logarithms.

Crash course in logarithms (to isolate t or τ):

If *e* is raised to a power, take the natural log of both sides:

$$e^{-t/\tau} = 0.26$$

$$\ln(e^{-t/\tau}) = \ln(0.26)$$

$$-t/\tau = -1.35$$

Use calculator's ln button.

Rabbit hole to next slide!

Why 63.2%?

Substituting τ for t in the voltage equation, gives:

$$V_C = V_0[1 - e^{-t/\tau}]$$

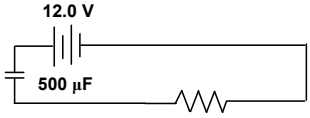
$$V_C = V_0[1 - e^{-\tau/\tau}] = V_0[1 - e^{-1}] = V_0[1 - \frac{1}{2.718}]$$

$$V_C = 0.632V_0$$

A capacitor is "fully charged" after "several time constants."

2. Voltage Example

What will capacitor voltage be after 0.150 seconds?



$$\tau = RC = 550\Omega \bullet 5.00E-4F = 0.275s$$

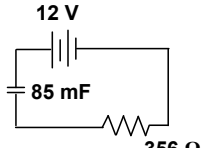
$$V_C = V_0[1 - e^{-t/\tau}]$$

$$= 12.0V[1 - e^{-0.15s/0.275s}]$$

$$= 5.05V$$

3. Energy Example

How much energy does the capacitor store after it has charged for 85 seconds?



Find time constant:

$$\tau = RC = 356\Omega \bullet 85mF = 30.3s$$

Then voltage:

$$V_C = V_0[1 - e^{-t/\tau}] = 12V\left[1 - e^{-\frac{85.0s}{30.3s}}\right] = 11.3V$$

Finally:

$$U_C = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2} \bullet 0.085F \bullet (11.3V)^2 = 5.43J$$

Discharging Capacitors Through Resistors

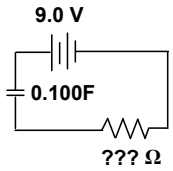
As a capacitor discharges through a resistor, voltage diminishes logarithmically:

$V_C = V_0 e^{-t/\tau}$	V_C = Capacitor Voltage V_0 = Battery Voltage t = time (s) τ = RC (s)

4. Flashy Example

A flash camera uses a 9.0 volt battery to charge a 0.100 F capacitor through a resistor
(Ever heard that high pitched sound from a camera? That's the sound of a capacitor charging!).
What is the resistance if the capacitor needs 80% of max. voltage in 5.00 seconds?

t = 5.00 s



Flashy Answer

First, 80% of maximum voltage is 7.2 V.

Next: $V_C = V_0[1 - e^{-t/\tau}]$

$$7.2V = 9.0V[1 - e^{-5.00/\tau}]$$

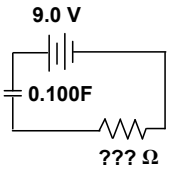
$$\frac{7.2V}{9.0V} - 1 = -e^{-5.00/\tau}$$

$$0.2 = e^{-5.00/\tau}$$

$$\ln(0.2) = \ln(e^{-5.00/\tau})$$

$$-1.609 = -5.00/\tau$$

So: $\tau = 3.11\text{s}$



Finally: $\tau = RC = 3.11s$

$$R = \frac{3.11s}{0.100F} = 31.1\Omega$$

Homework

3.C.5 Problems.
Due: Next Class.

Finish Unit 3.C Review Problems
Due: ???