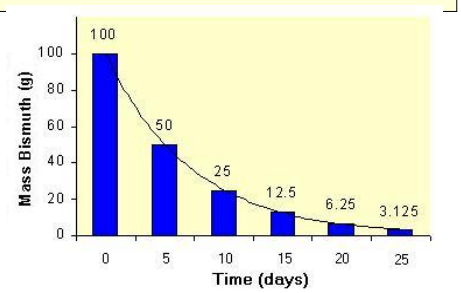
6.6 - Half Lives



Activity

As a radioactive material decays, the parent isotope diminishes at some rate, which depends only on the isotope in question.

The activity of a sample is measured in decays per time unit (USUALLY a second).

The SI unit is the becquerel (Bq):

1 Bq = 1 decay/second

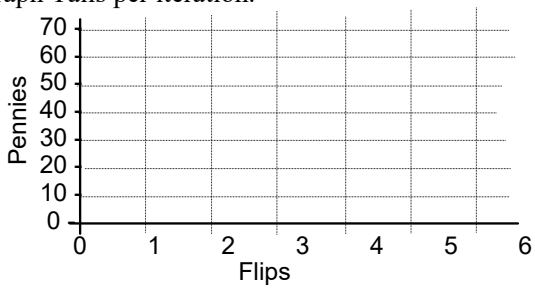
1. Penny Activity Activity!

Count your pennies.

Flip them - remove Heads and count remaining Tails.

Repeat, and again, and again, until tails are gone.

Graph Tails per iteration.



2. Activity Example

A sample has an activity of 44.4 MBq, and is studied for 24 hours.

If each decay produces 1,500 eV, how much energy (in eV) will the sample give off?

Calculate total decays:

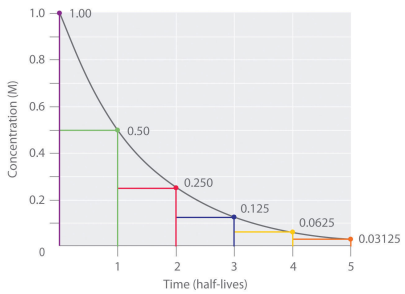
$$44.4E6\,Bq \bullet 24\,h \bullet \frac{3600\,s}{h} = 3.84E12\,decays$$

Total energy (eV):

$$3.84E12\,decays \bullet \frac{1.5E3\,eV}{decay} = 5.8E15\,eV$$

Half Life

Decay rate is often reported in half lives, the time required for a sample to lose half its initial abundance.



3. Half Life Example

1.0 gram of a radioactive sample with a half life of 2.5 days sits for 10.0 days. How many grams of the original isotope remain after this time?

10 days represents 4 half lives, so half of a half of a half of a half is $1/2^4$, meaning that 1/16th of the original mass remains: 0.0625 grams.

Bludgeon method proof:
after 2.5 days = 0.5 grams remain
after 5.0 days = 0.25 grams remain
after 7.5 days = 0.125 grams remain
after 10.0 days = 0.0625 grams remain.

Activity Math

It is useful to determine the decay constant (λ) for an isotope before figuring out numbers of undecayed or decayed atoms in a sample.
Based on the mathematical definition of activity:

$activity = \left\lvert \frac{\Delta N}{\Delta t} \right\rvert = \lambda N_0$	activity = decays/time unit ΔN = number of decayed atoms Δt = time (units) λ = decay constant (time unit)$^{-1}$ N_0 = initial number of atoms

and calculus, half life and decay constant relate thusly:

$t_{\frac{1}{2}} = \frac{0.693}{\lambda}$	$t_{1/2}$ = half life (time unit) λ = decay constant (time units)$^{-1}$

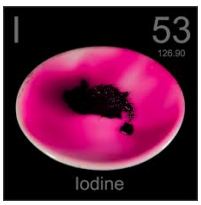
Atom Population

Once the decay constant is known, one can determine isotope populations for a sample at some time in the sample's lifetime:

$N = N_0 e^{-\lambda t}$	N = number of undecayed atoms N_0 = original number of atoms λ = decay constant (time unit)$^{-1}$ t = time (match units)

4. Atom Population Example A

The half-life of iodine-131 is 8 days (690,000 s).
At t = 0, 4.0 E 14 nuclei of I-131 are in a sample.
What is the sample's activity at that time?



Decay constant:

$$t_{1/2} = \frac{0.693}{\lambda}$$

$$\lambda = \frac{0.693}{6.9E5s} = 1.0E-6s^{-1}$$

$$activity = \lambda N_0$$

$$= 1.0E-6s^{-1} \bullet 4.0E14 atoms = 4.0E8Bq$$

5. Atom Population Example B

How many nuclei remain after 1.0 day (81,250 s)?

$$N = N_0 e^{-\lambda t}$$

$$= 4.0E14 nuclei \bullet e^{-1.0E-6s^{-1} \bullet 81{,}250s}$$

$$= 3.7E14 nuclei$$

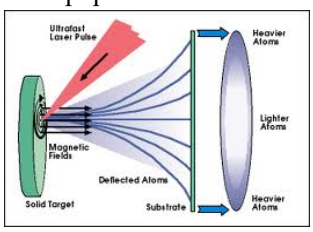
Radioactive Dating

Radioactive nuclides can be used as clocks.

Previously, decay math determined a future nuclide population.

Working backwards, one can determine the age of a sample based on a present population.

Isotope Enrichment Gadget



Radiocarbon Dating

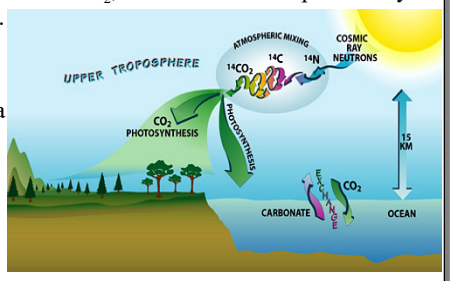
Uses C-14 ($t_{1/2}$ = 5730 y) in an organic sample, (bone, wood, etc.), and is reliable up to about 50,000 years.

Atmospheric C-14 abundance is 1 : 7.2 E 11 atoms C, and is continuously produced by cosmic ray bombardment.

Plants uptake carbon as CO_2; when animals eat plants they uptake C-14 also.

Sample C-14 diminishes over time through beta decay.

$${}^{14}_{6}C \rightarrow {}^{14}_{7}N + {}^{0}_{-1}e$$



6. Mummy Example

A sample of mummified tissue from a tomb contains carbon-14 nuclei that have decayed to 18% of their original value.
How old is the sample? (C-14 half life = 5730 years)

How old is this mummy?

Radiocarbon Answer

18% means that 18 C-14 atoms remain out of 100.

Find the decay constant from half life (in years^{-1}):

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{5730\,y} = 1.209\,E-4\,y^{-1}$$

Solve for t: $N = N_0 e^{-\lambda t}$

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$\ln\left(\frac{N}{N_0}\right) = -\lambda t$$

$$\frac{\ln\left(\frac{N}{N_0}\right)}{-\lambda} = t = \frac{\ln 0.18}{-1.209\,E-4\,y^{-1}} = 14,200\,years$$

Radioactive Dating Applications

Other isotopes are used to date much older things.

Potassium-argon (K-Ar) dating is used in geochronology. At the time of crystallization of a mineral, there's some amount of K-40 (half life = 1.2 billion years) present. K-40 breaks down into Ar-40 by electron capture: ${}^{40}_{19}K + {}^{0}_{-1}e \rightarrow {}^{40}_{18}Ar$
Since argon is larger than the spaces in the crystal lattice, it remains trapped in the rock. Determining the K-40:Ar-40 ratio allows for age determination.

Uranium-lead dating is similar, with a half life of 4.47 billion years. U-238 decays into lead-206.

Other Radiation Units

Activity doesn't give a complete picture of health risks: different types of decay affect biological systems differently.

The sievert (Sv) is a stochastic (having probability-driven effects) measure of long-term health effects (such as cancer) for low doses.

The gray (Gy) is a deterministic (having single-outcome effects) measure of immediate health effects (such as tissue damage) for high doses. 1 Gy = 1 J/kg.

The sievert takes into account the type of radiation, as well as exposure mode (inhalation of radioactive dust, external exposure, etc.).

Homework 6.6

Problems 6.6 in your Booklet
Due: Next Class