

AP Phys 2 Unit 6.7 Notes - Mass-Energy Equivalence

6.7 Mass - Energy Equivalence

Energy points to E, mass points to m, squared points to c², equals points to the equals sign, and speed of light (constant) points to c.

Mass-Energy Equivalence.

In particle accelerators, matter often attains speeds approaching that of light, and relativistic effects come to bear.

Kinetic energy of moving objects at relativistic speeds is different than at slower speeds.

Classical vs. Relativistic?

The question arises: when does one use relativistic effects to calculate energy?

Rule of thumb: classical works well when objects travel $< 10\% c$: kinetic energy error $< 1\%$.

A Little Time-Saver: γ (Resources P. 9)

Relativistic equations often use a coefficient to adjust energy calculations.

The symbol is γ (not gamma ray!), and is always greater than or equal to 1.

You have a resource, but the equation is:

$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$	$v = \text{object velocity (m/s)}$ $c = \text{light speed (} 3.0 \times 10^8 \text{ m/s)}$
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Relativistic Kinetic Energy.

As objects travel faster, kinetic energy increases.

As they reach relativistic speeds, kinetic energy no longer follows classical physics:

$K = \frac{1}{2}mv^2$ <p>Classical</p>	$K = \text{kinetic energy (J)}$ $m = \text{mass (kg)}$ $v = \text{velocity (m/s)}$
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vs.

$K = (\gamma - 1)mc^2$ <p>Relativistic</p>	$K = \text{kinetic energy (J)}$ $\gamma = \text{gamma}$ $m = \text{mass (kg)}$ $c = 3.00 \times 10^8 \text{ m/s}$
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1. Kinetic Energy Example

How much more kinetic energy would a proton have traveling at 50% the speed of light ($0.50c$) using classical physics, vs. relativistic physics?

Proton Mass = $1.67 \times 10^{-27} \text{ kg}$

Classical: $K = \frac{1}{2}mv^2 = \frac{1}{2} \cdot 1.67 \times 10^{-27} \text{ kg} \cdot (0.5 \cdot 3.0 \times 10^8 \text{ m/s})^2$
 $= 1.88 \times 10^{-11} \text{ J}$

Relativistic: $\gamma = 1.15$

$K = (\gamma - 1)mc^2 = (1.15 - 1) \cdot 1.67 \times 10^{-27} \text{ kg} \cdot (3.0 \times 10^8 \text{ m/s})^2$
 $= 2.25 \times 10^{-11} \text{ J}$

Difference:

$2.25 \times 10^{-11} \text{ J} - 1.87 \times 10^{-11} \text{ J} = 0.38 \times 10^{-11} \text{ J}$

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Relativistic Momentum

Likewise, momentum must account for relativistic effects:

$p = \gamma mv$	p = momentum (kg m/s) γ = gamma m = mass (kg) v = velocity (m/s)
Relativistic	

vs.

$p = mv$	p = momentum (kg m/s) m = mass (kg) v = velocity (m/s)
Classical	

Relativistic Total Energy

So far you've considered total mechanical energy to be the sum of potential and kinetic energy:

$$E = K + U$$

Einstein showed that at relativistic speeds energy is:

$E = \gamma mc^2$	E = Energy (J) γ = gamma m = mass (kg) c = 3.00 E 8 m/s
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Mass-Energy Equivalence

From the equation on the previous slide, you can see that if an object is stationary, *it still has energy*.

This gives rise to Einstein's famous equation:

$E = mc^2$	E = Energy (J) m = mass (kg) c = 3.00 E 8 m/s
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This is called **mass-energy equivalence**: mass is in actuality a form of energy.

Conversion of mass to energy doesn't happen willy-nilly, but significant conversion of mass to heat energy does happen in nuclear reactors.

2. Mass-Energy Example

How much matter would need to be converted to energy to power a nuclear submarine (average power output = 48.0 MW) for 30 days?

Find energy in joules:

$$E = 48.0 \text{ E } 6 \text{ J/s} \cdot 30 \text{ days} \cdot \frac{24 \text{ hours}}{\text{day}} \cdot \frac{3600 \text{ s}}{\text{h}} = 1.24 \text{ E } 14 \text{ J}$$

Then shift equation to isolate mass:

$$E = mc^2$$

$$m = \frac{E}{c^2} = \frac{1.24 \text{ E } 14 \text{ J}}{(3.00 \text{ E } 8 \text{ m/s})^2} = 1.38 \text{ E } -3 \text{ kg (1.38 g)}$$

Not much!

Homework 6.7

Problems 6.7 in your Booklet
Due: Next Class